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Toward a Functional Theory of Politics

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Toward a Functional Theory of Politics submitted by David Alexander in partial fulfilment of the requirements for the degree of Master of Arts.

Abstract

This study is an attempt to contribute toward the construction of a political system model formalized in the functional mode, by attempting to specify the elements from which such a theory could be constructed.

To this end, the study consists primarily of a review of relevant literature, directed towards outlining the requirements of formal functionalism and towards garnering such formal structure and conceptual material as can be turned to the purpose of a functional political theory. Following an introductory discussion of functionalism, there is a review of politically related formal literature: 'Social Choice Theory', 'Game Theory' and some formalized material from substantive political science, discussed in chapters 2, 3 and 4, respectively.

The study concludes with a summary of the results of the review, an outline of a preliminary functional model, a 'conceptual sketch' of certain 'essential variables' of politics, and a discussion of the possible future construction of a functional theory of political action.

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I. INTRODUCTION: TOWARD A FUNCTIONAL THEORY OF POLITICS

A. Purpose of the Study

Eventually I should like to construct a political system model[1] formalized in the functional mode. The present study is merely an attempt to specify the elements from which the theory could be later constructed.

To this end, the study will consist primarily of a review of relevant literature. The review will be directed towards outlining the requirements of formal functionalism and towards garnering such formal structure (and conceptual material) as can be turned to the purposes of a functional political theory.

The study will be organized into the following basic sections: an introductory discussion of functionalism, a review of politically related formal literature, and a concluding summary of results with respect to the contruction of a functional theory of politics. I shall discuss functionalism in this chapter. The most important bodies of politically related literature are 'Social Choice Theory' and 'Game/Coalition Theory' along with some formalized material from substantive political science. I shall discuss these areas in chapters 2, 3 and 4,

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[1] More specifically, I shall be considering electoral/parliamentary systems, which could be seen as only a subsystem of the political system. However, since this is the focus of my interest in viewing the total political system, I shall refer to this system as *the* 'political system'.

respectively. In chapter 5, I shall summarize the results of the review, set out a preliminary functional model, outline a 'conceptual sketch' of certain 'essential variables' of politics, and discuss the possible future construction of a functional theory of political action.

Before beginning the discussion of functionalism, however, I shall detail briefly the points discussed and arguments made in each section of the study.

It is important that a social science which aspires to be theoretical explicitly avail itself of the rich and varied structures of mathematics and logic. In chapter 1, I place the proper use of mathematics in the context of scientific theory, outline some global divisions among the variety of mathematical structures, and select *functional formal systems* for further consideration. The special formal characteristics and advantages of functional explanation are discussed and the manner of proceeding in using functional mathematics in a functional theory is outlined. These points are elaborated in a brief history of functional mathematics and of the development of functional theories in physics and economics.

I next discuss the *'functional tradition' in social science*. I argue that the major themes and concerns in this tradition are particularly suited to the use of functional mathematics, but in fact it is not so used. The major problem in the functional social science literature is a metaphorical elaboration of concepts and statements, which

elaboration results from the failure to explicitly and rigorously engage in mathematical reasoning. Functional mathematics is appropriate for this literature, and some formal characteristics do obtain, but social science (excepting economics) has not developed a formal functional theory. These points are illustrated in a short discussion of T. Parsons and D. Easton, along with some work in biology which does combine conceptual concerns similar to those of social science functionalism with an attempt to apply functional mathematics.

Following the discussion of formal and 'traditional' functionalism, I review the relevant formalized literature in a search for formal and conceptual material amenable to functional treatment. In **chapter 2**, I discuss the *theory of social choice*. I argue that social choice theory offers a formalized definition of politics as a method of aggregating individual choices to a social state: this definition is embodied in the classic work of Arrow. The definition also includes formalized political concepts, particularly the axiomatized 'democratic conditions', and is partly functional in nature. I examine the adequacy of the social choice model of politics by discussing some violations of the model in real systems, the incomplete framework for individual action assumed in the model, and the lack of consideration in the model for some clearly important aspects of politics.

In **chapter 3**, I discuss *political game theory*. I argue that game theory allows the formalization of clearly crucial aspects of politics. The basic game theoretic constructs, conflict of interest, partial control of outcomes, and strategic interaction, are of obvious political relevance. And the rich variety of game types which have been elaborated by game theorists offer further formalizations of political concepts. Among these, I argue for the particular importance of the N-person game and its political interpretation, 'coalition theory'. In coalition theory the basic concern is the formation by political actors or parties of coalitions; the basic problem is the search among all possible such for stable (or 'equilibrium') coalitions. I discuss a number of equilibrium solutions in the literature and argue that one can discern a trend from the sole consideration of power or weights as coalition determinants to the additional concern with policy (issues). One such, with the formal and conceptual characteristics of an essential variable, is the 'minimum winning coalition' (Riker, 1962) with ideological constraints (Axelrod, 1970), for which empirical support has also been adduced.

In **chapter 4**, I discuss some material from the substantive political science literature in which aspects of politics have been formalized; these offer alternative formalizations for some of the concepts already discussed. I discuss first a geometric or *'spatial' analysis* of electoral competition which includes, among other ideas, a

geometric formalization of 'conflict of interest'. I next discuss several articles which offer alternative formal approaches to the possibilities of *'power and policy success'* in voting bodies. These include a statistical argument for the importance of 'bloc' voting, a statistical defense of factions, and a graph theoretic formalization of 'political balance'. I argue that the import of these articles is their support for the evolution of decision-making bodies into well disciplined blocs or parties. In this, they add formal support to the earlier arguments for the importance of the game-theoretic 'minimum winning coalition'. The final group of articles discussed in chapter 4 deals with the distortion in elections between the votes cast for parties and the number of seats obtained by the parties; that is, the *'vote/seat' relation*. In these articles are formulated mathematically such political properties as plurality versus proportional electoral systems, the stability of two-party systems, and the rise and fall of third parties. These are complementary to social choice theory, offering alternative formalizations of the aggregation of individual choices into a social decision.

In **chapter 5**, I conclude by first summarizing the results of my review of the literature. I then discuss the possibility of construction of a formal functional theory; this discussion is in terms of the requirements of formulating essential variables, the creation of 'special'

functional theories explaining key aspects of politics, and the ultimate construction of of a 'general' theory of political action.

Although no such general theory of politics has been accomplished, I do present a preliminary *special theory*, in which I define an essential variable, subsystem variables and their relations, and attempt to show that the variables act to maintain the essential variable at an extremum value. I then outline a *conceptual sketch* of other variables which could be candidates for treatment as essential variables in special theories and in a general theory. I then briefly discuss the possible future theory of political action and finish with some concluding remarks.

B. Functional Theory

In order to place the study theoretically I shall now attempt to clarify the expression 'functional theory' as here understood and then to contrast this with familiar functional approaches in social science (see pp. 22-33).

Formal Functionalism

Formal functionalism consists in employing for explanation a particular family or mode of formal systems. It will, therefore, first be necessary to define the concept 'system', state its significance and note distinguishing features of formal systems.

Systems: the general language of science

In the view taken here the concept 'system' may be used to provide a general characterization of scientific discourse. The definition is as follows:

> A system is a *set*, partitioned into two subsets. The first consists of *variables*; the second consists of *relations* on the variables.[2]

Variables and relations

Whatever the substantive focus one does science through the formation and study of variables and relations; that is, systems (Ashby, 1956; McGinnis, 1965;[3] Mesarovic, 1975). The 'systems' concept and its elaborations provide a general language for scientific discourse. Such discourse is carried on at several levels, embodied in types of systems which differ in their formal and semantic characteristics.

Types of systems

Three system types are particularly relevent here: formal, conceptual, and theoretical (Jung, 1974). In *conceptual* systems concepts (semantically meaningful variables) are related to produce propositions, the whole

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[2] (Jung, 1974) The above is embodied in the basic formal language of set theory. This purely formal definition reflects the view that a general language of science should be a minimal language (Jung, 1976; Mesarovic, 1975); embodying as little initial structure as possible, so that in further elaboration structure is explicitly added rather than assumed. It also avoids any implication of 'real' systems. This discussion of systems, including the 'families' of formal systems, follows Jung (1971, 1973a, 1974, 1976).

[3] 'Social sciences ... the theory of such fields is made up largely of two component parts, variables and relations.'(p. 4)

system outlining an area of inquiry.

Formal systems are simply the systems of mathematics and logic, systems of tautologies with no conceptual or empirical content. Such formal systems serve science as sources of uninterpreted structures upon which to draw in relating conceptual variables (Kemeny, 1959).

Given only conceptual variables all relations are possible: the concepts alone give only a 'possibility space' in which the relations must be located or specified. In science one specifies the relations between concepts both empirically and formally. Formally, conceptual systems are specified by being mapped into some formal system. This further process of relating a formal system and a conceptual system to each other produces a *theoretical* system, the variables being constructs and the relations laws (Jung, 1974).[4]

The 'subject matter' of the theory is at the level of the conceptual system - economic theory is economics and not physics because of its concepts, even if the formal systems may be the same, as in Newtonian mechanics and economic equilibrium analysis. Conversely, one may have the same concepts handled differently in a formal sense, as in static versus dynamic economics.

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[4] Empirically, conceptual systems are specified by operationalization (operational definitions) for the purpose of experimentation (hypothesis testing).

Types of formal systems

In theorizing, the particular formal systems by which one specifies the relations among one's concepts are as crucial as are the concepts themselves and may be considered separately.

Indeed, historically, formal and conceptual systems have variously influenced each other. Non-Euclidian geometry developed as a formal system; when applied to the concepts of physics it led to a theoretical revolution. Conversely, in economics game theory as a formal system was originally developed in response to particular 'substantive', 'meaningful'; that is, conceptual or theoretical, problems (see Ch. III).

Given the formally separate and historically varied influences between particular formal systems and particular conceptual systems, the choice of a formal system is at least partly arbitrary. Yet, it is crucial, and mistakes are made: it is necessary to justify the formal system which one chooses to adopt.[5]

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[5] Poor theory may be ascribable to three general problems: inadequate conceptualization, inadequate formal manipulation (whether derived from poor choice of system or illogical manipulation), and inadequate handling of the relations between conceptual and formal systems.

The failure to clearly distinguish formal from conceptual systems and to consider each level on its own terms commonly results in degeneration to a sort of 'theory by metaphor' in which formal and conceptual meaning are mixed up rather than clearly related (see pp. 25-29). For example, Henderson (1935), in discussing Pareto's work on the Social System, commends his use of analogies to 'physico-chemical' systems but cautions that it is not the properties (concepts) themselves but the application of a logical method (mathematics) which has been found useful in

[5]

Level of formalization

One may distinguish formal systems according to level of formalization and according to scope. Differing levels of formalization are exemplified in the classification commonly made into ordinal, interval and ratio variables (Stevens, 1959); and in the varying degrees of structure in (for example) algebraic systems such as rings and groups.[6]

The usual mistake is to assume too much structure too early: to choose (out of familiarity or for reasons of computability) highly structured systems (e.g. the calculus) with all their 'strong assumptions' about variables and relations, and thereby fail to, in the words of Ashby (1956), 'allow simple answers to simple questions'.

Scope

With regard to scope the common error is to choose some particular narrow structures (perhaps selected from another discipline) and fail to avail oneself of the varied relations possible with a wider formal scope.[7]

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[5](cont'd)all sciences.

[6] Felix Klein (Courant, 1964) has classfied geometry according to formal properties maintained under transitions: one gets point set theory, topology, projective geometry, affine geometry, euclidian geometry; that is, levels of formalization.

[7] The recent efforts in 'catastrophe theory' have been criticised for this, for using these theorems without knowledge of the wider scope (topology) within which they must be seen (Zahler, et. al., 1977). This is also true of various 'engineering' approaches to systems (see Berlinski, 1976). For a positive example, see the formalization of Heider's theory of 'structural balance' in graph-theoretic terms, and the subsequent use of this area rich in theorems to derive the 'structure theorem for signed graphs'

[7]

Given the above, the better strategy at the level of formal systems is to deliberately avoid closure: to include material at varied levels of formalization and to deliberately place one's formal approach within a wide scope.

While wanting to avoid a narrow scope, the absurd extreme of expansion to all of mathematics is no solution. Between the extremes one may involve substantive (conceptual) as well as formal criteria for a choice: mathematical areas may be viewed a general solutions to types of problems, formal systems appropriate to the demands of certain subjects (with both conceptual and empirical features). Thus, while a 'mechanics' of politics may be too narrow, one may aim for a wider scope more adequate to the 'nature' or 'meaning' of politics in the 'game theory' of politics (Riker, 1962). Or economic theory develops under the demands of the subject from 'perfect market' to 'oligopoly' theories.

I will below claim for formal functional systems special 'adequacy' for handling a political functional theory.

Families of formal systems

Given the foregoing considerations, I should like to outline a division of classical mathematics, with particular consideration to use in theorizing, into four global modes

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[7](cont'd)(Ch. IV-C).

or families of formal systems. These are deterministic, functional, comparative-genetic, and probabilistic (d'Abro, 1951; Jung, 1973a; Nagel, 1956; Stinchcombe, 1968) These four families are formal systems for different types of theories: they also allow of certain differences in theoretical and methodological approach.

Deterministic theories are appropriate to theoretically closed and empirically isolated contexts. Variables are all treated alike. The mathematics is differential calculus. In theories formalized deterministically, it is required to establish some 'initial state' of the system; that is, the values of all the variables at some time. Once this is done, it is possible to generate all other states of the system, forward or backward: there is no 'history' in such a system; prediction and retrodiction are equally possible (D'Abro, 1951).[8]

Functional systems are appropriate to the explanation of action: of systems maintaining some of their characteristics while varying others over certain situations. Variables are partitioned into three subsets. The mathematics is the calculus of variations and developments from it.

Comparative-genetic systems are appropriate to contexts in which the 'history'[9] of the system is a necessary

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[8] An example is Newtonian planetary astronomy.
[9] Or, more accurately, one must formally allow for *change* in the *structure* of the system over time (genetic) or in some other comparative context.

consideration. The mathematics is integro-differential calculus.

Probabilistic systems are any of the above with one or more variables having a probability distribution over its values.

My aim is a functional theory of politics in the above sense.

The Functional Formal Mode

I have outlined above four general families of formal systems upon which to draw in theorizing. I should like to elaborate upon the basic formal properties of functional formal systems; then briefly outline the development of formal functionalism; first its purely formal development in mathematics, then its use in particular substantive areas.

Functional variables and relations

As mentioned, a deterministic system uses the differential calculus (in classical analyses) to examine systems which can be assumed to be theoretically closed and empirically isolated, and for which the variables can all be treated alike.

In contrast, a functional analysis does *not* assume isolation of the system. In fact, the logic required is specifically for the analysis of a system which maintains some variable (or returns to it) in the face of environmental disturbances. Such theories are in physics called 'action' theories (D'Abro, 1951).

Nor are the variables of the system all treated alike. They are partitioned into three subsets: an essential variable or variables (Ve), functional subsystem variables (Vs), and boundary condition variables (Vbc)(Jung, 1978a). One's interest is in the relations between these sets of variables: between the essential variable and the functional subsystem variables, and, secondarily, between the functional subsystem and the boundary condition variables.

The essential variable: *Ve*

Conceptually, the essential variable is one which is some defining characteristic of the system; some important, 'essential' feature of the area of concern: without it the phenomenon would, in some sense, not be the same phenomenon. In a 'control-theoretic' design problem the essential variable would be 'what is to be controlled'; in a 'naturalistic' theory it would be what *is* controlled or maintained; in the 'preliminary model' (Ch. V) it is the variable 'power', formalized as a minimum winning coalition, along with the others offered as candidates in a functional formulation of politics. In general, an essential variable could be any social 'constraint'; legal, rational, practical, and so forth. This idea is wider than the concepts 'purpose', 'goal' or 'homeostasis', commonly considered essential in social analysis.

The key *formal* requirement for formal functional analysis is that the essential variable be maintained at an *extremum* (maximum, minimum, or constant). This is necessary

for the application of the logic of formal functionalism.

The functional subsystem: *Vf*

Given the essential variable, the search is for some system of variables, the relations among which are such as to maintain the essential variable at an extremum. One examines all the variables and relations in the *Vf*: a small change in one should result in a countervailing change in others so as to maintain or return the essential variable to the extremum.

Boundary condition variables: *Vbc*

Here are specified relations to the environment. One treats these as a set of separate variables: they are not part of the system. The concern is to ascertain under which values of these variables the *Vf* will still be able to maintain the *Ve* at an extremum, and under which it will not.

Formal functionalism

Formal functionalism is meant to refer to the *explicit* use of a functional system as a formal system. Functional theory then requires an attempt to conceptualize the area of inquiry with the explicit formal constraints in mind. The first task of theory in this mode is the selection of variables which are essential, in both the conceptual and formal senses (see Ch. V).[10]

The political system theory to be developed is intended to be functional in this strict sense; that is, in the

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[10] Variables which are *empirically* essential, in that they approximate extremum values or fluctuate about equilibrium points, will be of particular interest.

explicit use of a functional formal system in addition to the functional character of its concepts. In this respect it would differ from most current functional theory in social science(see below, 'The Functional Tradition').

Classical formal functionalism: the calculus of variations

Classical formal functionalism is a branch of the integral calculus called the Calculus of Variations. The calculus of variations developed out of the consideration of extrema (maximum, mimimum and stationary values) of various types of functions in differential calculus. Out of this consideration there came a new type of problem, first posed in a contest by Jean Bernoulli in 1696 (Smith, 1929; Forsyth, 1960; Petrov, 1968) called the 'brachistochrone (shortest time) problem'.[11]

It required, not the analysis of values of a set of variables for which a function is at an extremum, but the seeking of a curve (function) along which the value of an *integral* is at an extremum. This integral is not a function of X in a straightforward sense; rather it is a function of an entity *itself* dependent on the function and its derivatives (Allen, 1938). The integral depends upon the entire function.

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[11] It was required to find a particular path (function) between two points a and b (the domain), down which a mass falls under the influence of gravity in the shortest time (the integral - here a minimum). This method was quickly turned to other problems, some very old (e.g. the Greek 'isoperimetric' problem).

The fundamental problem of the calculus of variations is to find the *function* which, integrated over *some given domain*, produces an *extremum of an integral*.[12]

Approach to a variational problem

One has some known pattern of change, expressible as an extremum. One needs to find the 'primitive' function(s) which, under small variations, will maintain this extremum. One generally has to build in boundary conditions to get a specific function. The problem is thus one of extremizing an integral subject to constraints - a theory of 'action' (D'Abro, 1951).[13]

Extensions of the classical theory

From the early problems there has developed a rich and varied literature of formal structures, dealing with a wide variety of problem types and solution strategies.

L. Euler (1707 - 1783) developed a single method for solving the early problems. By reducing the integral to a series of arcs, he was able to achieve solutions trough the better-known functions of a large number of variables.

J. Lagrange (1736 - 1813) devised a specific method, called the 'variational method' for dealing with variational problems, and solved more complex functions; for example the

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[12] In terms of the previous discussion, the function to be found is the structure of the functional subsystem Vs; the integral is the essential variable Ve; the given domain is the set of boundary condition variables Vbc (along with other variables necessarily held constant).

The integral was in 1903 given the name 'Functional' (Monna, 1973); classical calculus of variations is now considered to be part of the General Theory of Functionals.

[13] For examples, see Petrov (1968), Tauber (1969).

extrema of multiple integrals.

Weierstrass (1815 - 1897) developed theorems for weak and strong extremals, and for extremals which are discontinuous.

Volterra (1887) developed the study per se of all functions which depend on all the values of another function (as the extremal function depends upon all the values of the *Vf* variables): this led into the General Theory of Functionals.

Consideration was extended to *systems* of variables through the development of the 'state-variable' approach (Tauber, 1969), which expresses a system of variables in some space by means of a single variable in a higher-dimensioned 'phase/configuration' space.[14]

More recently, there was a sudden expansion of interest in problems in the calculus of variations with the development of aviation. Variational methods were created for the determination of 'optimal control' laws, for rockets and later for wider engineering applications. This led to optimal control theory, with its new methods and problems in the design of control systems.

Among further developments was dynamic programming, in which the extremal integral or optimal path was replaced by a piece-wise 'optimal policy': best decisions within a restricted framework (Bellman, 1967).

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[14] This method was that used in Gibb's thermodynamics, which was, in turn, the format in which the 'theoretical functional' analyses (to be discussed) are couched.

In addition to classic extrema one may be required to find extremal probability functions, extremal *risks* under incomplete knowledge or opponent's strategies (see Ch. III), or optimal objective functions under constrained choices of methods.[15]

Formal functionalism in Physics

I have above outlined the formal development of functional systems; that is, the development within mathematics. I should now like to briefly mention formal functionalism as it has been substantively developed in relation with particular conceptual systems; specifically, those of physics and economics.[16]

The first problems in the calculus of variations were also problems in physics, and all along there has been a 'formal functionalism' in the above defined sense (concepts formalized in a functional formal mode) in physics.

These were, however, minor in the corpus of physics as a whole. For, after some very early functional formulations (e.g. in optics) there was a general rejection of them. With the replacement of Aristotle's 'vis a tergo' by Galileo's 'inertia', teleological explanations of all sorts were rejected in physics, and so also did a 'formal teleology' remain undeveloped (Frank, 1948). And, with the

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[15]These latter go beyond strict functionalism as above defined; that is, they go beyond considerations of functional stability to 'feedback' and other control system properties.

[16] I shall ignore functional analyses in such other areas as psychology (action theory) and philosophy.

success of Newtonian planetary astronomy the accepted model for theories in physics was 'deterministic' or 'mechanistic'.

The decline of mechanism

But in the 19th century, particularly in the context of attempts to reconcile Newtonian mechanics and classical thermodynamics, functional theories were again developed. In this 'decline of mechanism' (D'Abro, 1951; Wiener, 1943), embodied in Gibbs' thermodynamics and Wiener's theory of comunications, there developed functional theories which were soon deemed especially appropriate to the 'teleological' problems in biology and social science (Frank, 1948).[17]

Formal functionalism in economics

In economics the development is from equilibrium analysis (differential calculus) to classical optimization problems in which a stationary optimum is found (integral calculus, game theory) to dynamic optimization (calculus of variations, optimal control theory, dynamic programming), in which an *optimal path* is to be found: this last is formal functionalism (Chaing, 1974; Tauber, 1969; Petrov, 1968).

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[17] In modern physics both Quantum and Relativity theory exhibit a functional character (Jung, 1981), the speed of light in a vacuum 'C' being the formalized extremum in the latter case. And in modern Cosmology, some have offered in place of deterministic analyses of cosmic evolution the 'anthropic' principle, in which 'possible universes' are constrained by the necessity to arrive at a condition congenial to human observers (Gale, 1981).

Formal functionalism has been a part of economics for some time (Allen, 1938). For example, S. Krupp (1965) discusses the use of 'mechanistic' and 'teleological' equilibrating systems in economics, and, in a manner similar to Nagel (see below), recommends to other social scientists the use of these methods of economic analysis.

Functionalism in Social Science

As noted above, the development of functionalism in mathematics occurred through the drive to solve new types of formal problems, both within mathematics and in close relation to theoretical problems in physics.

The Functional Tradition

In social science, the functional tradition derives from an almost primordial concern with purpose and goals. This 'conceptual' functionalism, whether in natural philosophy or latterly in biology and the humanistic social sciences, has been driven by a 'meaningful' concern with purpose, goal orientation, coherence, and the vitality of natural or social life.[16] As such, the functional approach has been widespread in the social sciences (Merton, 1957).

There is no intrinsic relation between the term 'functional' in its purposive or 'teleological' sense and the use of 'functional' to describe the calculus of variations or the theory of functionals.

Yet they are found together in ways which suggest that consideration of both traditions is indicated in the development of a 'Functional Theory' (see '**Formal & Conceptual Origins of Functionalism**' p. 23). Historically,

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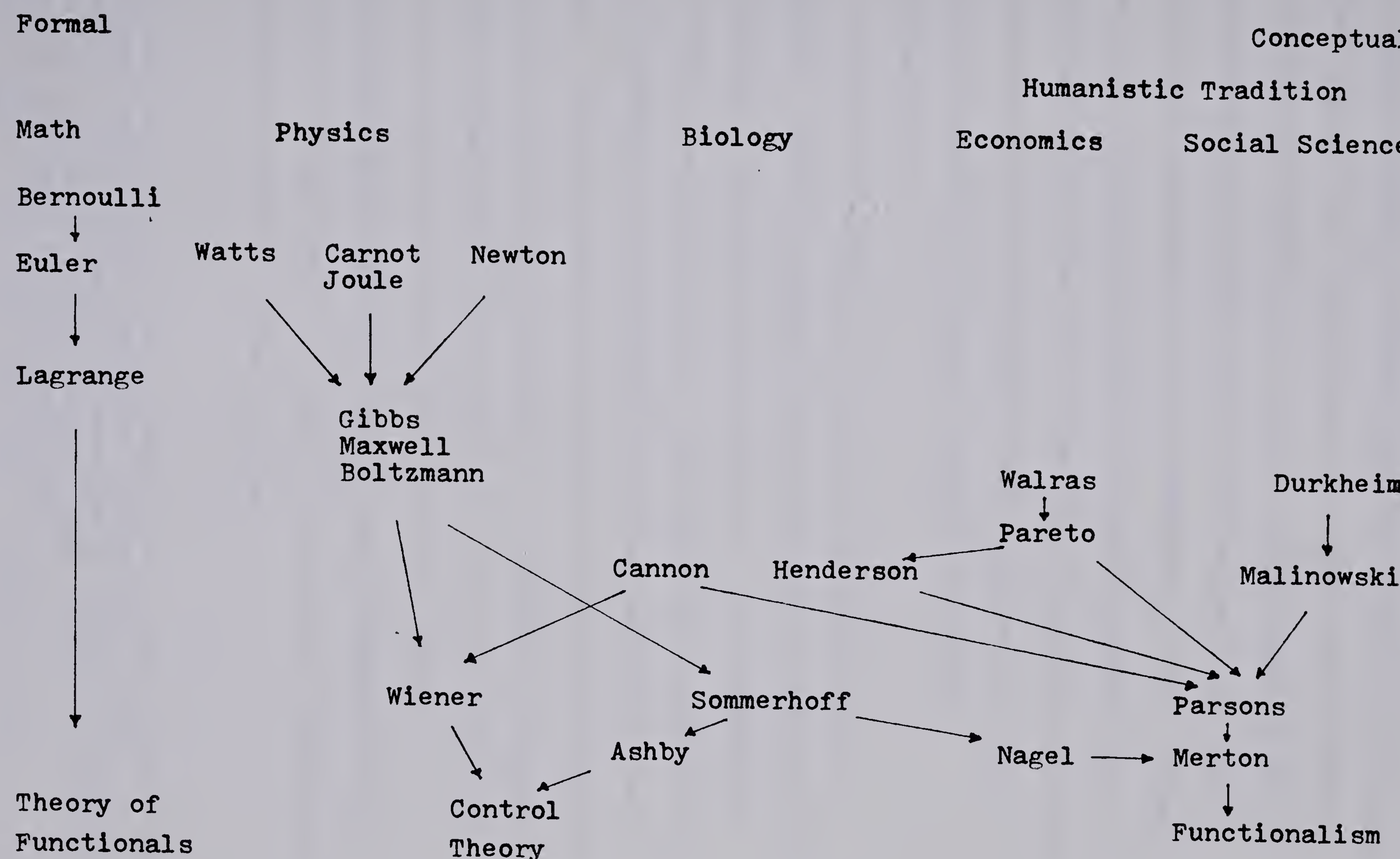
[16] This was once true of physics, as can be seen in Hero's optics and Maupertuis' Principal of Least Action (a functional interpretation with God as cosmic economizer) (D'Abro, 1951). However, the 'mechanistic' tradition prevailed until the late 19th century, and even with the 'decline of mechanism', the resulting functionalism was formal (Wiener, 1943). Biological functionalism is discussed with 'social' science, as economic functionalism was earlier in formal functionalism.

Formal & Conceptual
Origins of Functionalism



both appear together in early (and some modern) physics as somehow appropriate (as previously noted); and in much of the functional tradition in social science *some* of the formal features of formal functionalism are found (see '**Partition of Variables in Functionalism**', p. 28). Also, formal and conceptual variables are found together as theoretical constructs. My contention in placing these two in parallel is that the proper formal approach to the functional tradition in social science is formal functionalism.[19]

I shall not attempt a review of the functional tradition. Since my concern is to develop a theory within the framework of formal functionalism, I am concerned to note formal characteristics;[20] the functional tradition in social science is of value mainly on the conceptual level. I shall, therefore, not be interested in detailed analyses of the concepts of functionalism.[21]. Rather my interest is in those formal distinctions which do exist, in those who do deal with formal and theoretical structures (see 'Theoretical Functionalism', below), and in demonstrating that much of functionalism in social science is inadequately formulated in terms of formal systems

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[19] An alternative functional formulation to the calculus of variations would be the tradition, going back to the Scolastics, which attempts to develope a 'logic of purposive action' or 'teleological calculus' (Jung, 1981).

[20] And any concepts formally embodied.

[21] Which have in any case been done. (Merton, 1957; Buckley, 1967)

Theory by Metaphor/Analogy

Among functionalists there are those who clearly and explicitly work with formal systems, and even contribute to their development (e.g. economic game theory). But much functionalism represents a metaphorical approach to theory as distinct from formal functionalism. Starting from some theory or 'system' from some other field (biology or latterly cybernetics), this approach ignores the rigorous logical treatment necessary and engages in the mere elaboration of concepts of the theory. Many of the sociological, anthropological, and political functional theories are of this type. The work *is* theory in the defined sense: both formal and conceptual variables have some influence on the course of the theorizing. But the absence of formal rigor results in that metaphorical proliferation of concepts so characteristic of such endeavours. To the extent that formal constraints have been explicitly employed at all, they have been largely analogies from biology. The major contributions of social theorists in social science functionalism have been the introduction of more explicit logical elements into the analogies (see Nagel, 1962).

Theory by metaphor: science in the 'context of discovery'.

A. Koestler has speculated that the essential aspect of creativity consists in the bringing together, the clashing, of two different 'planes of thought' (1964). This

he holds to be true of all creative thought. Certainly it is true of science.

In a similar vein, the concepts of 'metaphor' and 'analogy' have been invoked to explain the 'artistic' or 'conjectural' aspects of scientific activity. That is; metaphors and analogies, because of their power to in some sense direct thought, are seen by many as major tools or speculative instruments in scientific theory-finding.

Power to direct thought

This might be considered a modified version of linguistic determinism, the view that linguistic systems exert influence over conceptual systems. Max Black, in *Models and metaphors* (1962), exemplifies it in outlining what he calls his 'interactionist' view of metaphors. Black states that, in any metaphorical statement, there are really two distinct subjects; a 'principal' subject and a 'subsidiary' subject. The metaphor forces one to connect the two. It does so by forcing one to use a system of implications inhering in one subject as a means of selecting and organizing relations in a different field. For example, if one were to attempt to describe a battle, using for this description the vocabulary of chess, the choice of the 'chess metaphor' would lead some aspects of the battle to be emphasized, others to be ignored - it would be a general's battle.

Black is aware of the dangers of metaphor. Metaphor accomplishes a transition, it does not demonstrate a

similarity.

In terms of the foregoing 'systems type' distinctions, the primary subject (biology or cybernetics) has both formal and conceptual characteristics. Both are applied to the secondary subject (society) but in a non-logical manner, rather than developing conceptual systems and formalizing them *explicitly*.

'Exemplary' in this respect is 'General Systems Theory', which is conceptually much wider and which differs in explicitly advancing metaphor as a method of theory construction (Lee, 1969). Here, the systems idea is a virtual 'world view', applied to all of reality.[22]

Partition of variables

However, the major formal distinctions (the partitioning of variables into three sets) have been present in functionalism from the start; that is, there has always been some distinction between essential variables (called goals, purposes, etc.), functional subsystems, and boundary condition variables (see '**Partition of Variables in Functionalism**' p. 28).

On the other hand, the critical formal constraint (that the dependent variables become formally essential only if

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[22] There is, within the general systems approach, the practise of a sort of 'theory by analogy' or modelling, in which it is recommended to look for *isomorphic* formal structures (Rapoport, 1968; Berlinski, 1976). This still seems not to allow for the richness and development of formal systems: in formal analogy one is simply comparing structures apart from their formal 'context'. Rather, real formal work within the formal system is necessary (Farquharson, 1969).

The Partition of
Variables in Functionalism

Pareto (1916)	Malinowski (1936)	Parsons (1951)
-functional prerequisites -internal forces (interests, residues, derivations) -external forces	-vital function -manner related within system -system relations to surroundings	-functional prerequisites -system structure -environment

Sommerhoff (1950)	Nagel (1951)	Ashby (1956)
-focal condition -system (with directive correlation -coenetic variables	G -goal variables S(ABC) -system state variables E -environment	E -essential variable R -system responses D -environmental disturbances

Lipset (1960)	Easton (1965)	Stinchcombe (1968)
-generalized leadership -representational system -boundaries	-persistence -system -environment	H -homeostatic variables, consequences S -structure T -tension, disturbance

Economic Dynamics

-dependent variables
-independent variables
-exogenous variables

they are constrained to extremum values) has not been explicitly recognized by the social scientists. The dependent variables are either treated formally as final (teleological) variables or as functions of an argument without the formal constraint of an extremum.

I should now like to discusss briefly the functional and systems tradition in sociology and political science.

Functional Sociology

T. Parsons: the social system.

The primary figure in sociological functionalism was Talcott Parsons. In his 'The Structure of Social Action', he addressed himself to the work of Thomas Hobbes and the utilitarian tradition. Hobbes' work was an extended metaphor of the 'mechanistic' physics of his day, and Parsons' concern was to remedy the rationalist deficencies of the utilitarian tradition by extending the scope of variables needed for adequate explanation of social action - hence his value theory.

The social system

It was, however, in the next 'phase' of his work that he undertook a program of functional theory construction (Martindale, 1960). 'The Social System' is an attempt to apply the 'system' idea to society, using the 'structural-functional' approach (derived from Radcliff-Brown).

The social system is one of the three ways in which social action is structured. And if a social system is able

within itself to meet the 'essential functional prerequisites' (empirical self-subsistence, duration, biological recruitment, socialization) it is a society.

Parsons and Pareto

Important in this phase is the relation of Parsons to an earlier sociologist, V. Pareto. According to Henderson (1935), Pareto's work in economics and sociology was the continuation of a long tradition of applied mathematics, a tradition including the work of Euler and Lagrange, the mathematics of equilibrium in elastic solids, and, more directly, the mathematical economics of Walras. In particular, the model of his work can be found in the thermodynamics of W. Gibbs, which work constitutes a general formal description of a physico-chemical system (and was functional in nature).

Pareto has done likewise for the social system, says Henderson. His sociology grew out of his economic system by the addition of variables which were necessary to encompass the manifestations of social heterogeneity and non-rational action.

But the key for Henderson of Pareto's contribution is his method. He replaced cause and effect analysis by a method of analysis involving the 'simultaneous variations of mutually dependent variables'; that is, of systems. And this social system is analogous to a physico-chemical system, not of the *properties* (concepts) of the system, but of the application of the *logical method* which has been

found useful in all sciences.[23]

With respect to Pareto, Parsons had, in 'The Structure of Social Action', drawn upon the non-rational elements for his value theory, and derived the idea of 'system' as a general approach to theorizing.

And in his 'Toward a General Theory of Action', Parsons discussed action (personal, social, cultural) as being organized into systems which are constrained through orientation to goals. The systems so constrained exhibit boundary maintenance and a tendancy to maintain order; that is, interdependence among the components of the system - these properties are true of all systems.

But in 'The Social System', Parsons used the idea of system, not as a 'logical method' but as a generalized metaphor for social theory. Crucial here was his turn (Parsons, 1937)[24] to the work of W. Cannon and in particular, Cannon's 'fruitless ... attempts to draw substantive analogies and homologies between biological organisms and social systems'(Merton, 1957).[25]

In Parsons' 'The Social System' we see the conceptual proliferation attendant upon his metaphorical approach, and this continues in the later phases. With the vocabulary of

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[23] Also, implicit in Pareto's work is a functional approach (Samuels, 1974). See also Ch. III.

[24] His 1968 Introduction, p. X.

[25] Merton refers to 'The Wisdom of the Body' (1932). Cannon's work does however show the partition of variables into three subsets and the order of doing a functional analysis as a general mode of formulation: establish certain functional requirements, then describe arrangement(s) through which these functional requirements are met.

systems, functions, and, in later work, cybernetics, he has produced a corpus which is general but not logically rigorous.[26] Indeed Parsons' work, while successfully remedying the mechanistic metaphor of Thomas Hobbes, has itself become an extended metaphor of modern physics and biology.[27]

Functional Political Systems

In political science most of the relevant literature consists either of metaphorical functionalism or of formal literature which is not explicitly functional. The former includes the work of Almond (1960), Lipset (1960), Easton (1965a,b), Luhmann (1974) and others. The latter includes formal social choice theory. Game theory seems to be divisible into areas which are explicitly functional and areas which are not so, though perhaps amenable to functional uses.

I shall consider social choice theory and game theory in chapters II and III respectively. Here I shall briefly mention work similar to sociological functionalism, in

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[26] The formal partition of variables is there, however. In 'The social system', the 'structural-functional' approach is the structure of the system, and the functional prerequisites of its maintanance. And in 'Economy and Society', the same partition can be seen in the four subsystems, operating together to maintain the four societal functions. One of these is the political system, for Parsons the subsystem 'Polity', which has the primary societal function of attaining collective goals (1966). (see also Lipset, below).

[27] Illustrative here might be comparison between Parsons and N. Wiener. Both had the same concerns for a holistic, non-mechanistic, 'humanistic' approach to theory; and both derived part of their orientations from the thermodynamics of W. Gibbs and attended Cannon's seminars.

particular the work of D. Easton.

Easton's Political System

David Easton is the pre-eminent functionalist in political science. In a project extending over 20 years and four volumes, his work shows typical features of functionalism in social science; that is, conceptual richness and proliferation, theory by metaphor (here the 'systems' idea), and the partition of variables.

Like the general systems theorists, Easton calls explicitly for analogizing as a method. In his case, the metaphor seems to be what is in control theory called the 'cannonical closed loop feedback system' (Berlinski, 1976). Easton sees the political system as an open feedback system.

Easton, like the other metaphorical functionalists, observes an implicit functional partition of variables in setting up his system: there is the system, the environment and the idea of persistence.[28]

Formalistic Functionalism

Here would be included much of 'mathematical systems theory', The exercises are purely formal. Much of game

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[28] S. Lipset, in his 'Party systems and the representation of social groups', (1960) maintains that a large part of what Parsons calls the Polity (the function of generalized leadership for the society) can be seen to fall within a 'fairly coherent' system of action, which he calls the 'representative system'.

The structure of the representative system derives from those institutional practices (especially parties and interest groups) which channel interchanges between 'authority' and the 'social groups' of a society. The boundaries of the representative system are this authority system (the legal and government apparatus) and the solidary groupings (religious, ethnic, etc.).

theory also is like this. The work includes various types of formalization, and should therefore add to the richness of the formal functionalist systems, but, according to Berlinski (1976), much of the work is lacking in logical coherence. In any case, most of the material is not strictly functional.

Theoretical Functionalism

It has earlier been acknowledged that the scientific use of formal structures is their combination with conceptual systems to construct a formal theory.

There has been in biology and related areas a tradition which is concerned to enhance theory by giving their conceptual concerns a more sophisticated formal structure (Sommerhoff, 1950, 1974; Ashby, 1952, 1956; Nagel, 1956, 1961, 1979; Isard, 1969; Ackoff & Emery, 1972; Wright, 1976; Woodfield, 1976).

These approaches are 'theoretical' rather than 'metaphorical' or 'formalistic' because they are explicit in their attempts to apply formal functional properties (versus, for example, Easton) and they are also explicitly concerned to preserve the 'humanistic' or 'purposeful' meaning of biological and social science.

For the most part, they are not *doing* functional theories; they are rather engaged in 'meta-theoretical' work: in discussions of the possibility of functional theories, of appropriate and inappropriate substantive applications, of criteria for choosing functional formal

systems over some other, and of definitions of functionalism itself.

G. Sommerhoff: Analytical Biology[29]

Sommerhoff's work is an attempt to characterize that 'directiveness' or 'purposiveness' or 'goal-orientation', which is the essence of biological behaviour, in mathematical terms. He explicitly uses formal functional systems in this attempt. In a 'state-variable' approach, he identifies goal-directed behaviour, not with some vitalistic phenomenon nor with 'consciously purposed activties of rational agents' (1974: 17), but with the *formal* 'system property', directive correlation

Directive correlation

The end or purpose is formalized as a goal variable or variables (or more accurately the subspace of their extremum values) which he calls the 'focal condition'. Directive correlation gives the focal condition a certain independence or stability, by maintaining it in the face of variations or disturbances in environmental (coenetic) variables. This maintenance Sommerhoff calls functional equifinality.

In the course of his study, Sommerhoff offers formalizations of such important biological concepts as adaptation, regulation and integration. He considers he has demonstrated that the essence of organic life can be as formally and clearly stated as any concept in physics. Thus he has superseded both the felt necessity for a

[29] (1950; 1974).

'mechanistic' approach, and the 'ideational' or 'rational' implications of the usual treatments of goals and purpose. And his pioneering effort to develop an 'objective teleology' has led others to attempt a formal approach to social action.

R. Ashby: The organism as machine[30]

Following Sommerhoff, Ashby outlines the 'logic of mechanism', formalizing a number of biological concepts. These are done in terms of a cybernetic and control/automata theoretic formalization. He clearly defines, conceptually and formally, the idea of 'essential variable';[31] sets out the logic and method of formal functionalism (largely as stated earlier: p.13-19);[32] and formally outlines stability, adaptation, and other 'control' processes.

Sommerhoff and Ashby develop their logic and method of formal functionalism in the context of the biological sciences and provide many illustrations of its potential application to a variety of problems. Such formalized problems and solutions as, for example, the development of subsystems to allow adaptation to greater amounts of environmental disturbance are clearly of importance in the

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[30](1952; 1956).
[31] 'Every species has a number of variables which are closely related to survival and which are clearly linked dynamically so that marked changes in any one leads sooner to marked changes in the others' (1956: 42)
[32]The essential variable(s) *E* and a set of variables *R* embody certain responses which the system can make, in the presence of environmental disturbances *D*, which act to maintain the essential variables in some critical interval (1956: 36-48, 82, 224).

application of formal functionalism to social science.

E. Nagel: Formalization of functionalism[33]

Following both Sommerhoff and Ashby, Nagel explicitly addresses the functional tradition in social science, as embodied in the survey of functionalism by R. Merton (1957). Using their state-variable approach, he endeavours to 'make explicit the elements ... involved in any directively organized (or functional) system'. In his terms the 'elements' are a system *S* and its environment *E*. There is also some trait or traits *G*, dependent upon *S*. These elements are expressed formally as state coordinates. In a system directively coordinated with respect to some *G*, alterations in *E* or *S*, which would otherwise cause *G* to 'fatally' change, are compensated for by other changes in *S* so that *G* remains.

Nagel then uses this formalization of functionalism to analyse Merton's 'codification' of functionalism, and to set out the necessary requirements for a formal functional theory.[34]

The debate over functional statements.

There have been, since Nagel's work, several attempts to apply the formal structures to 'purposeful systems'

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[33] (1956).

[34] This involves examining the the items in Merton's paradigm for functional analysis, and attempting to relate them to features of formal functionalism. For example Nagel stresses the need for specifying *formally* those features which could be 'functions'.

However, Nagel's call for a sociological functionalism which would explicitly work within a formal functional framework was unfortunately ignored.

(Ackoff & Emery, 1972) or 'self-organizing systems' (Formby, 1965).

There has been as well a more 'philosophical' approach, in which a number of workers (including Nagel) have debated functional 'issues': definitions of functionalism, appropriate substantive applications, including the basic question of whether there is *ever* any advantage in a functional approach, or, indeed, if there *is* any genuinely functional analysis (Nagel, 1979).

C. The Study and Formal Functionalism

This study is preliminary to the construction of a formal functional theory of politics. As mentioned (p. 15) the first task of such construction would be the identification of particularly salient features of politics and the formulation of them as essential variables; that is, variables that are both formally *and* conceptually essential.

The conceptual analysis of politics which would be required for such a task is beyond the scope of this study. I shall restrict myself to those formal structures which can be extracted from the literature and an evaluation of their usefulness to a formal functional theory.[35]

An attempt will be made to distinguish the formal approaches by formal criteria; that is, according to the amount of structure embodied in the variables and relations (or 'levels of abstraction'; Mesarovic, 1975) and the types of structures utilized (deterministic, functional, comparative-genetic; Jung, 1973a).

Also, approaches will be distinguished according to the nature of the subject matter, 'nature' referring again not to conceptual but to formally consequential features (e.g. small-N combinatorial or large-N probabilistic approaches).

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[35] However, at least a 'conceptual sketch' of the political system, including preliminary salient variables, will be offered, including such candidates for treatment as essential variables as power, policy, legitimacy, enforcement and sovereignty (see Ch. V).

II. THE THEORY OF SOCIAL CHOICE

A. Introduction

The Theory of Social Choice consists of a large and varied body of formalized literature. A mathematical approach to social choice obviously has considerable relevance for any attempt to develop a theory of electoral systems; for votes, referenda and other such phenomena are no more than the making of choices in the context of a larger social system. Indeed, the theory of social choice constitutes a virtual definition, in mathematical terms, of electoral systems.

In addition, the social choice literature includes in its formulations a number of formalized political concepts (theoretical constructs), including some that could be turned to use in a functional formalization.

In this section I shall outline briefly the dual origin of social choice theory in the mathematical study of electoral and committee rules beginning with Borda and Condorcet (Black, 1958), and in welfare economics and its utilitarian precedents. Then I shall describe in general terms the social choice formalization of political systems. I shall follow this with a more detailed discussion of the 'classic' work in social choice theory; that is, K. Arrow's *Social choice and individual values*, which is viewed here as an 'ideal' or 'theoretical' political system. This will be followed by a discussion of the adequacy of this 'ideal'

system in the context of real political systems. In the final section I shall comment briefly on the functional possibilities of social choice theory.

The Mathematics of Politics

The application of mathematics to political choice had its beginning in the French Enlightenment. In that formalizing and optimistic period the transition 'from natural philosophy to social mathematics' (Baker, 1975) was embodied in the work of J-C de Borda (1737 - 1799) and the Marquis de Condorcet (1743 - 1794).[36]

Borda

In 1781, Borda presented for consideration an arithmetic example of an 'election', in which there were 21 electors and 3 candidates, and in which he showed that a single vote might well result in the election of a 'wrong' candidate. For, supposing that 8 of the 21 electors were to rank candidate A over B & C, 7 rank B over C over A, and 6 rank C over B over A; then, even though a majority (13) of the 21 electors preferred B *and* C to A, a vote would select A the winner.

Borda suggested two ways out of the problem, both of which attempt to take account of the 2nd and 3rd ranked choices of the electors. In one he suggested a series of elections, in which candidates would run two at a time. In

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[36] The following account follows Black (1958).

another, he suggested an attempt to assign candidates relative 'degrees of merit': given an elector with a ranking of A over B over C, one would specify a 'degree of superiority' of A over B and of B over C. Specifically, he suggested making the degrees equal:[37] representing C's merit by a, B's merit will be a+b, A's a+2b. He further imposed a numerical value, letting a=b=1. The election so conducted would give candidate A 37, B 42, and C 47 'units of merit', and *now C* would be elected.

Borda's work marked a beginning, and suggested further courses one might take in a mathematical theory of elections. Focussing on what is now called the *preference relation*, his suggestions were first attempts to go beyond 'ordinal preference'. His two-by-two elections prelude the variety of electoral laws (including pair-wise elections) which real political systems have developed. And Borda also claimed some wider application for his method: that it would hold for any sort of group (corporations, committees) and any type of ranked choice.

Condorcet

Condorcet, in his 'Essai' of 1795 (Black, 1958), approached the theory of elections from a probabilistic perspective. He considered three candidates A, B and C, and any number of electors. He then demonstrated that, in a single vote system, a candidate *might* be chosen who would in

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[37] In modern terms, he had imposed an equal interval difference on an ordinal preference relation.

fact lose in a vote against each of the others separately. He recommended in such a case that any winner should have to get at least half the total votes cast.

Even so, there might fail to be a choice. So he recommended that voters be called upon to give the *order* or ranking of the candidates. Each voter would be required to express his preference in three pair-wise propositions: (A over B) *or* (B over A); (A over C) *or* (C over A); and (B over C) *or* (C over B).

The Condorcet paradox

Given this, Condorcet showed that choosing the majority proposition from each pair would result in eight different rankings of the candidates; that is, eight different sets of three majority propositions. Six of the eight seem to leave no doubt who is the best choice of the three; for example, majority choices for A over B, A over C, and B over C for the three sets would give the total result A over B over C.

Two sets, however, leave the 'best' candidate in doubt, since they are inconsistent. In one set, the three majority propositions are A over B, B over C, and *C over A*. In the other set, they are B over A, A over C, and *C over B*. This type of 'inconsistent' result has come to be called the *Condorcet paradox*[38]

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[38] Such situations would later be called 'cyclic majorities'. Black points out that calling such situations 'inconsistent' is rather inappropriate, since it seems to require at the level of the voting results an order (or consistency) that can really only be expected of individuals (see Arrow below).

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In the six consistent cases Condorcet recommended the criterion that, if out of the three candidates one could get a majority over each of the others, he should be the winner. This is in spite of the fact that, in certain cases of large majorities, the probabilistic approach leaves the 'best' candidate in doubt.

For the inconsistent cases, Condorcet recommended a procedure which removes *one* of the three propositions in the set: the one which has the lowest majority. Choosing the winner from the other two is relatively straightforward.

Condorcet then extended his analysis to cases of any number of candidates. In such cases the importance of the inconsistent sets loomed much larger. And it became clear to later workers that the problems of real interest lay in finding the treatment of these 'contradictory' cases.

Condorcet was further able to show that any criterion which required a voter to give a definite value to each candidate in his ranking (such as Borda's 'degree of merit') could be made, by judicious choice of example, to give ridiculous results (such as counting second choices higher than first).

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[38](cont'd)Black himself shows that it is possible to avoid inconsistent social choices (or 'cyclical majorities') if one can only impose on the electors what he calls a 'single peaked' preference. Such a situation might occur in a real election if, for example, the candidates were representative of some ideological spectrum (see 'Spatial Models', Ch. IV).

In addition, Condorcet first raised as a problem the relative value between choice orderings - should one's choice of A over B be given as much weight as his choice of B over C?

The work of Borda and Condorcet introduced problems with which any formal consideration of social choice must deal, including the 'preference' relation, the problem of 'inconsistencies' in aggregating individual choices, and the problem of measuring 'degree of preference' (utility theory). Following their work, such men as Laplace and Dodgson have written on the problem of elections and committee voting. But, until Arrow's work combined collective choice with welfare economics, it remained a subject of little note.

Welfare Economics

Welfare economics has, according to Schumpeter (1954), a history going back to the Middle Ages. But the welfare economics of the modern era may be said to have developed from the work of Marshall. And, since the main features of this welfare economics are essentially a revival of the utilitarianism of Bentham, I shall first briefly discuss the utilitarian movement.

The Utilitarian backround

The backround of utilitarianism is the attempt, in the work of Hobbes, Locke and Hume, to emulate the sciences in

method and achievement.

T. Hobbes

For Hobbes (1588 - 1679), this was an attempt to develop a 'geometry of motion', which would offer a general 'mechanical' explanation of men's thought and action and of society.

His explanation started with a model of the individual: the individual is 'governed by passions'; his sole aim is to satisfy his desires. And though his '*ends*' are irrational, he does so by using rational *means*.

The relation of the individual to others was an extension of this individualistic and rationalistic approach. For the individual could not fail to see that his ends could only be achieved through others and that therefore he must achieve power over others. Thus, the problem of order in Hobbes' system.

D. Hume

In his individualism, and his rationality of means, but not of ends (passions, desires), Hobbes has been called the 'grandfather' of utilitarianism. And D. Hume (1711 - 1776) would be the father. Hume's general concern was to use the experimental method of reasoning to explain human behaviour by means of natural laws. In the course of applying his empiricist system to the ethical and emotional side of human nature, he established the basic category of 'moral approval', which he defined as an emotion produced by actions which are useful or agreeable to one; that is,

actions which have 'utility'. From this idea that 'general utility' is the cause of social approval Hume went to the strict utilitarian position that general utility is its justification. He thereby arrived at an ethical system based on utility.

J. Bentham

J. Bentham (1748 - 1832) was the utilitarian par excellance.[39] Bentham based his philosophical system on the proposition that the 'general welfare' was the only rational measure of value; indeed the only ethical measure, for in their action men are wholly moved by pleasure and pain.

Pleasure for Bentham was a homogeneous 'substance' a measureable and comparable magnitude. For its summation to the general welfare he proposed a 'felicific calculus'.

Economics and utilitarianism

There was a close relation between classical economics and philosophical utilitarianism. The 'rational individual' model at the basis of classical economics was utilitarian in nature, and its vision of collective rationality was this 'isolated optimizing individual' operating within a free-market system.

And classical 'welfare' economics - that part of classical economics concerned with the attempt to prescribe a method leading to the maximum social good (as seen in Marshall) - considered utility much as had Bentham: utility (or satisfaction, or welfare) was quantitative, and

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[39] The name 'Utilitarianism' was bestowed by J. S. Mill.

satisfactions of different people can be compared and in fact summed up into a general welfare for society as a whole (interpersonal comparability). Indeed, this was a basic criticism: the unreality of this 'cardinal' utility.

Pareto's contribution

V. Pareto, in the course of his evolving work, made several contributions which touch upon the above and related points.

In the context of the disrepute of cardinal utility, Pareto developed a measure which allowed comparisons *between* utilities (greater, lesser, or equal) and which still allowed maximization problems to be addressed. This formed the basis for 'ordinal' utility.

In welfare economics directly, he developed a definition for general welfare which has since become known as the 'Pareto optimum' (see part C, below). Actions are Pareto optimal which allow no member of society to be worse off than before the actions.[40] In this, Pareto's work was a real attempt to do formal functionalism, in the above defined sense.

Pareto also recognized the necessity to go beyond the rational model, in two senses. First, he recognized in his

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[40] Thus Pareto 'saved the day'(Schumpeter, 1964) for welfare economics by allowing the definition of a maximum without interpersonal utility.

The 'classical result' of welfare economics is that perfectly competitive markets allocate private goods Pareto optimally (Riker, 1973). Public goods, as distinct from private, are not so allocated (see Prisoner's Dilemma, Ch. III).

sociological work that there will in fact be mixes of different (densities of) actor types: the aggregate characteristics of these types will be crucial for any social decision theory. Second, under the influence of Nietzche (Samuels, 1974), he came to regard non-rational actor types as of prime importance.

Arrow's 'new' welfare economics

However, these later advances of Pareto were not to be a part of the tradition of subsequent welfare economics which underwent a revival with the work of K. Arrow. Arrow combined elements of the welfare economics tradition with a generalization of the Condorcet paradox. His work, and the responses to it, are modern social choice theory.

B. The Political System in Social Choice Theory

In the social choice view, political systems (including electoral systems and decision-making bodies) are formally defined as functions relating individual preferences to social preferences. All such systems are, in this view, methods of aggregating or mapping individual decision sets on some set of social alternatives (Murakami, 1968).[41] Theorists of democracy have considered how 'best' to do

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[41] 'A society consists of individual members and a society's decision is composed of its members' decisions. Each society has its own rule for making decisions. When a combination of individual decisions is given, a society's decision is reached according to that rule.' etc.(op. cit. p. 1) For other approaches to the aggregation problem, see Ch. IV.

this; that is, fair ways to relate social policy or objectives to the preferences or aspirations of the members of the society, and their respective orientations have been combined in Arrow's work. Before considering it I shall outline the general format.

Individual Choice

In social choice theory the approach is to embody certain substantive assumptions about individual actor propensities as formal constraints. The basic formalization is set theoretic.

Alternatives

There is assumed to be a set of choices or alternatives.

Let $A = (x, y \ldots z)$.

These could be of any sort; candidates, social states, policies etc.

Individuals

There are a number of individuals.

Let $I = (1, 2 \ldots n)$.

Preferences

It is assumed that each individual ranks or orders the choices with respect to each other; that is, imposes some relation on the set of alternatives - a preference relation.

The formal expression of preference can take a number of forms. The basis for all are expressed as properties of

binary relations between all choices taken two at a time, of the form 'xRy'.

Such properties are:

1. Reflexivity: for all 'x' in 'A', xRx. Every alternative must be as good as itself.
2. Transitivity: for all 'x', 'y' and 'z' in 'A'; if xRy & yRz then xRz. For example, if R= 'is taller than' then one being taller than a second who is in turn taller than a third, implies that the first is also taller than the third.
3. Completeness: for all 'x', 'y' in 'A', either xRy or yRx (assuming x and y are not identical). This means that the individual 'knows his mind' with respect to every alternative.
4. Antisymmetry: for all 'x', 'y' in 'A', if xRy & yRx then x=y. For example, the relation 'greater than' is antisymmetric.
5. Asymmetry: for all 'x', 'y' in 'A', if xRy then not-(yRx).
6. Symmetry: for all 'x', 'y' in 'A', if xRy then yRx.

The properties usually considered to apply to the preference relation in social choice theory are transitivity, reflexivity, and completeness: these together make an 'ordering' of the set of alternatives. However, other combinations are sometimes used; that is, the individual's preference relation may be a 'quasi-ordering', 'partial ordering', 'strict partial ordering', or 'strong ordering'

according to the formal properties satisfied (Suppes, 1957).[42]

The result is a profile of preference orderings, an N-tuple summarizing the individual preferences of the n individuals in the society.

Social Choice

The social choice is the 'outcome set'. It is generally assumed to be, like the individual preference set, an ordering over the set of possible social states, candidates, or whatever are the elements of the individual choice set. That this assumes, in some sense, 'rationality' in society has often been seen to make it an excessively unrealistic assumption: Black commented on the 'mistake' of Condorcet in regarding choices of candidates which failed to show ordering 'inconsistent'.

The Function

A social choice function (or social decision rule) is a function which associates each profile of individual preference orderings with some ordering for the society itself (the social choice).

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[42] All these are ordinal relations. Some studies would go beyond this to 'intensities' of preference, or to the comparison of intensities of preference.

In real political systems, social choice functions are embodied as voting procedures, constitutions, and such like. Some possible ways to do this are given specific names: consensus, where everyone must agree (and the obverse of which is an individual veto); methods of varying proportions necessary for a certain outcome (e.g. a two-thirds rule for constitutional change); methods where one individual can determine the social choice (individual initiative or dictatorship; or some method which treats all individuals alike (e.g. majority rule).

Many specific functions could be used to relate the individual choices to the social choice: often methods in actual use are criticised for failing to satisfy some desireable criteria (see Proportional/nonproportional systems, Ch. IV), So, the typical method is to impose certain restrictions on the function, restrictions which embody some valued criteria of a social choice system, for example, 'equality' in majority rule.[43]

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[43] The social choice literature begins with a formalization and attempts to approach real systems from there; for similar work, but starting with real political systems and attempting to formalize, see Ch. IV.

C. An Ideal Political System

The institutional embodiments of social choice systems with special relevance for politics are electoral systems and decision-making bodies. I shall here discuss a classic work (Arrow, 1963) relevant for both political contexts. This particular example of an ideal or theoretical political system will then provide the context in which alternative formal approaches and the departures of real systems can be discussed.

K. Arrow: Social Choice and Individual Values

Arrow's work is in the tradition of welfare economics. It relates directly to Pareto, but goes back to the utilitarian economists.[44] Also, it combines this tradition with the formal theory of elections, in particular generalizing the 'Condorcet' or voting paradox. The general problem is the *fair* amalgamation of individual interests or choices into some social decison. For Arrow, this is the combination of individual preference patterns into a social preference pattern.

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[44] In some ways Arrow's work and that deriving from it (the 'new' welfare economics and theory of social choice) represents a regression from Pareto's work. The narrowness of the choice/decision model compared with Pareto's expansion into non-rational elements, and the failure to consider varieties of individual actor types and that the mixes of different actor types would be essential for understanding collective decision, represent a stage beyond which Pareto had soon advanced.

The social welfare function

Certain 'reasonable' democratic values are formally embodied in the system by imposing restrictions on the function and on the domain and range. These 'democratic values' are imposed on the choice set and the function as formal constraints in addition to the 'rational' assumptions. Taken together they make his Social Welfare Function (SWF). In addition to those 'rational' properties (transitivity, completeness, and reflexivity) mentioned earlier, Arrow sets four 'value' conditions on the alternatives and the function.

The choice set

1. Unrestricted domain. The scope of individual orderings from which to choose must be wide enough to allow any logically possible set of individual orderings: there should be no restriction on reasonable freedom of choice.

The function

2. The 'weak Pareto' principle. If every individual prefers one alternative to another, then the social choice must show the same preference. This condition assures that the social ordering will respond positively (or at least non-negatively) to changes in individual values.
3. Independence from irrelevant alternatives. The social choice over a set of alternatives must depend upon the orderings of individuals only over *those* alternatives.

4. Citizen sovereignty/non-dictatorship. The method of summation is such that the social outcome is not imposed by one person. This outlaws the choice of some social policy which is unrelated to the preferences of the members of the society.

The general possibility theorem

Arrow demonstrates that, given these restrictions, the system is contradictory; that is, there is *no* social choice function which satisfies all the axioms. This holds apart from any particular method of summation. This negative conclusion has led to a great deal of work since Arrow (see Sen, 1969). Among the major themes are the development of other formal and value conditions, the effects of relaxing one or more of the conditions, the generalization of the social choice model, and the relations of real choice situations to the formal conditions and results.

D. Real Political systems

As previously noted, Arrow's work constitutes a model or an 'ideal' political system, in that certain democratic virtues or values are formally embodied as conditions by imposing restrictions on the functions and the related sets. It should be clear that in real political systems these 'ideals' will be violated.

I shall below assess the adequacy of the social choice model as a definition of the political system. This will include the noting of violations of the conditions in real

political systems and a discussion of the social choice model of individual choice in the context of a wider framework of political actors and actor types; that is, the 'problem' of rational action. I shall also discuss the adequacy of the social choice definition of the political system and its place in the larger society in the light of a preliminary outline of the 'essential variables' of politics [45].

Violations of Arrow's conditions in real systems

The reality of the model of individual rational choice per se will be discussed below: here I will discuss the rational properties of the preference relation and more particularly the 'democratic values' which Arrow has imposed on his choice set and social welfare function.

Completeness

To 'know one's mind' over every choice combination requires comparability in the alternatives (Riker, 1972). This is unlikely in a complex political situation.[46] Nor is it necessary in real political systems - for political choices one needs only sufficient support or consensus over some partial range of alternatives to reach a decision.

Transitivity

This has been shown often not to obtain in the preference ratings of individuals. In the choice among political candidates it seems reasonable, perhaps less so

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[45] Ch. V.

[46] In committees, the rule of motion relevance is meant to ensure this.

among 'constellations' of policies.[47]

Unrestricted domain

This is a necessary condition for the paradox to hold. And it seems reasonable. Yet many real systems do violate this condition, such as majority rule in a decision-making body and run-off elections. These restrict freedom of choice by forcing choice (and the social differences underlying them) into a binary mode (Riker, 1972). Any traditional two-party system does the same, by custom if not by law.

And it has been shown that if one can only assume 'single-peaked' preferences in the population, the paradox doesn't hold. Such a situation seems likely: whenever the 'alternatives' (such as political parties) can be seen to be arranged in a single 'ideological spectrum', preferences will be single-peaked.

The 'weak Pareto' principle

This in effect asserts that there is some positive (non-negative) connection between votes and the social choice. This could be violated, if one were to count some votes negatively. More to the point this condition is compatible with virtually any state of society, however good or bad.

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[47] A related question to the foregoing is whether or not the *social outcome* needs to be 'rational' (transitive). In fact, it is not in real systems. It is necessary only to reach some decision, not to have a 'ordering' of the alternatives.

Citizen sovereignty/non-dictatorship

This rules out a 'dictator' outside or inside the system. This in fact is not as unlikely as it seems. Many countries have relations with others such that there is no real choice over certain ranges of policy. This condition would also be violated under the rule of some customary code, 'general will' or ideological rule which is considered to be beyond the scope of elective change (Riker, 1962).

Independence from irrelevant alternatives

This is meant to ensure that any choice which could win over each of the others individually will in fact win: this is also called the 'Condorcet criterion'. A rank-order weighted voting method sometimes used in selecting members to clubs can violate this condition (Arrow).

The Problem of Rational Action

Individual Assumptions in Political Theory

The formal approaches to political theory[48] commonly base themselves on a variety of assumptions about individuals, formally embodied as axioms. The basic set is probably decision theory; based on this are formulations of 'utility theory', 'rational choice theory' and the like. These assumptions concern not only cognitive orientation (e.g. ordinal, cardinal, interval measures) but also value orientations (e.g. the preference) and motivational factors regulating the transfer from decisions to actions

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[48]Including game theory as well as social decision theory.

(e.g. voting).

An adequate model of social choice would classify such assumptions, estimate their adequacy and outline other assumptions as alternatives or additions to those commonly used; that is, outline some 'possibility space' of assumptions regarding individuals; a space from which selections have in particular cases been made.

The problem in general is the relation of an aggregate theory to the level of individual actors or to a general psychology of political actors. While not suggesting that a comprehensive psychology must precede any attempt at an aggregate theory, the assumptions about individual propensities to act must be adequate to major sources of human conduct. In particular, as Pareto maintained, the actor model should include more than purely 'rational' components.

Cognition & Evaluation

As mentioned, the usual starting point for rational models is the 'nature of preference'. The givens are two: a set of alternatives and a relation on the set. This is an ordering or preference relation, at least to begin with (Suppes, 1957).

These givens precede consideration of both rationality and decision. The first assumption usually refers to the way the decision is made; rationality is not seen to depend upon level of knowledge (assumptions about knowledge), and which preference exists (however derived) is of no

consequence (rationality refers to means, not ends: Downs, 1957). One could handle rationally *any* space of alternatives under *any* evaluative constraints.

It would be better to consider many of the formally embodied assumptions (transitivity, cardinality) not as assumptions of rationality in any strict sense, but as assumptions of cognition or evaluation, and thus as prior to the question of rational decision or action. This is in line with the separation of questions of value (ends) and of knowledge (certainty, uncertainty) from the definition of rationality. One *could* act rationally given a variety of cognitive and evaluative states; for example, one could act rationally to kill oneself.

Action

One should also separate rational decision from *action*, strictly meant here as the accomplishment of the decision. Rational decison is intellectual, does not include action in the strict sense. Decision can rather be seen as setting up a 'propensity' to action (Rapoport, 1959). In a scheme which analytically separates knowledge, ends, decision, and action it is unclear where some of the assumptions are to be placed. One may consider the assumption of transitivity and/or reflexivity and/or completeness as part of the definition of rationality, or as conditions of rationality, which are strictly prior characteristics necessary for the application of rationality. Or, the ordering assumptions may best be conceived as part of rationality broadly

conceived, a sort of rational knowledge and value system.

The broad conception of rationality should not, however, rule out the consideration of alternative conceptions of rationality, formally embodied in different assumptions regarding knowledge and values.[49]

Rationality

All this would still be prior to the question of *rational* decision; that is, the independent estimation of probability and desireability, combining the two estimates in the approprite manner, and choosing the 'best' alternative (Jung, 1973). The broad conception of rationality and even some actual models fail to distinguish among (and thereby fail to partition assumptions among) knowledge, decision and action.

One theme in the discussion of rational models is the unrealistic precision in which the formal models are couched. The reason for this is the necessity to calculate, and this necessity comes from the particular modes of formalization theorists feels compelled to use. But given that 'most human reasoning is essentially shallow','rather than regard human reasoning processes as ... approximating to some refined and logical process' (Luce & Riaffa, 1957), one should formally embody this lack of precision. Recent developments in non-quantitative mathematics such as graph theory or fuzzy set theory (Gaines, 1976) would seem to

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[49]Nor for that matter formally embodied types of irrationality.

offer formal systems appropriate to imprecise human cognition.

The Aggregation of Action

Arrow and his economist predecessors assumed *a* model of the individual rational actor (however inadequate in terms of a theory of action): that there is some *one actor type* described by the formal axioms. In fact, as Pareto noted, in a society there will be a mix of different actor types and the aggregation of these will be crucial for real social decision. These types will not only differ in terms of rationality (or irrationality); there will be cognitive, evaluative and motivational types as well. Cognitively, there are well-informed and uninformed voters. There have been studies of the 'decision' to vote or not, and voters have been categorized as 'moral' or 'ethical' voters or alternatively as 'economic' voters.

The fact that such a mix of types exists has empirical implications. For example, rational decision models which make certain assumptions about individual action and then aggregate the results to observe whether the summation fits real results, must make gross estimates about densities or proportions of their assumed types before their summation makes sense. It would be useful to have an array of 'formalized actor types' to work with.

The mix of actor types will of course vary according to the social context. The individual is exposed to a myriad of social influences: choices will depend upon such features

of the political system itself as the party system and the electoral laws,[50] as well as the general social and cultural context.

The fact is that voters do not vote merely as isolated optimizing individuals, but as members of various groups in a particular society. The results for each person will depend not only on his own actions but upon the actions of others as well. That is, voters are in a 'game' situation, and some of the formal notions of game theory provide a necessary complement to the social decision model.

There have been some game-theoretic approaches to voting. Addressing the same problem as Arrow, R. Farquharson relates the Condorcet paradox to game theoretic considerations, bringing ideas of strategy and 'sophisticated' voting into an ordinal game-theoretic framework.[51]

The Political System in Society

In chapter 5, I have outlined several 'essential variables' of politics. In the light of these certain features of social choice theory become clear.

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[50] For example, Germans vote differently in the same election and for the same candidates according to whether their votes are summated by proportional or by plurality rules (Merkl, 1970). See also the 'wasted vote' thesis, Ch. IV.

[51] In a suggestive metaphor, Farquharson considers game theory as a model for the 'social contract', tracing the game-theoretic progression from a 'state of nature' to the formation of an assembly to the development of a party system.

Social choice theory is clearly about the aggregation of power and policy in a social context (see part E, below). The decision rules also embody certain important political realities, in particular, electoral norms. However, the important fact that political power is somewhat stable across a range of decisions is not reflected in social decision theory: the 'isolated individual' approach prevents this and other 'game' considerations from being formalized.

The political system operates within a framework of legitimacy. In real systems this often at issue. In social decision theory legitimacy is in some respects assumed ('feasible' alternative sets) and in others directly addressed: the formal constraints embodying democratic values.

In social choice theory enforcement is not considered: it is assumed. But decisions are not actions any more on the social level than they are for individuals: if the social decision process is to work, there must be some mechanism for translating law into action.

Sovereignty for individuals is directly stated as an axiom. In fact, this axiom is somewhat stronger than the situation in real political systems. For, while sovereignty is an essential aspect of a genuine political system (Crick, 1962), there are in practice limits on the scope of its operation.

E. Social Choice Theory and Functionalism

Social choice theory addresses itself to basic and traditional concerns of political rule. It does so in a way that formal structures are explicitly applied to these conceptual concerns; from it a large and rich body of literature has resulted in a number of formalized concepts (theoretical constructs).

The point of this review is not to give complete coverage but to gather formal structures and concepts for use in a functional theory. Given this, a tentative statement on some 'functional possibilities' in social decision theory will be given.

Formally, the utilitarian aspect of social decision theory is the one which seems most clearly to relate it to formal functionalism. The fact that social welfare functions are maximization problems suggests the functional nature of the work, and the use it could have in a formal functional theory. Indeed, it has been pointed out (Arrow, 1963, p. 110-111) that a social welfare function based on ordinal individual utilities will necessarily be a mathematical *functional* (see Ch. I), not a function in the usual sense: as a functional each social choice will depend on the entire individual ordering.

Furthermore, the 'social decision rules' which govern the result of a particular aggregation of individual

preferences[52] can be seen as institutionalized values. These values could be seen essential variables - as value constraints on social choice.

Alternately, the values could be viewed as boundary conditions (Vbc's) to the process of social decision. Unanimity in juries and 'citizen initiatives' for referenda can obviously be seen as embodying values particular to a certain social organization. The system produces one result under one Vbc, the same voting pattern produces a different result under a different Vbc. Much of the activity of politics could be seen as attempts to maintain some essential variable (power/policy) under particular institutionalized decision rules.

Social decision theory seems to show a duality of essential features which are aggregated in the process of decision making. These essential variables are here (see Ch. III) called 'power' and 'policy': the social choice view of politics clearly operates to aggregate both power and policy. This can be seen both at the level of the individual choice set, where the choice has both a weight or power component (+1 vote) and a policy component (the preference order), and at the level of the social choice, power being the results of the summated votes according to

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[52] For Arrow it is 'majority rule'. But there is a range of such, from 'individual initiative' (where one person's preference is enough to adopt some decision) through various proportions (five percent for a referendum, a majority, two-thirds for a constitutional change, all the way to unanimity.

some decision rule and policy being the content of the 'social states' chosen.[53]

One can as well plausibly interpret some of the departures from the value conditions in real political systems as features operating to maintain an essential political variable. For example, 'rational' order in the social choice is not necessary. What is necessary is that some winner with the power to decide be produced.

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[53] 'I am concerned with choices among social states ...and... a social state is a whole bundle of issues....The significant question ... is the existence of a majority on the bundle of issues represented by the candidate over any other attainable bundle.'(Arrow, 1963, p. 109).

III. POLITICAL GAME THEORY

A. Introduction

Game theory is a branch of mathematics which, like social decision theory, has been given an economic and political interpretation. Indeed, game theory is unique in that the theory was developed to address problems in social science.

It formalizes a number of relevant concepts and, like social decision theory, can be used to construct a definition of the political system. There has developed a large body of literature, thus ensuring richness in available formal structures. And there is also a growing body of work taking directly a 'game theoretic' approach to politics. Furthermore, game theory is, in some measure, a variety of formal functionalism.

I shall discuss the economic origin of game theory in the next section. This will be followed by a discussion of the essential formal characteristics of game theory and an elaboration of the characteristics of different types of games.

Further sections will deal with particular political applications of particular game types, especially political coalitions formalized as N-person games. I shall conclude with a discussion of the 'functional' nature of game theory.

B. The Economic Backround of Game Theory

Game theory has its origin in the attempt to go beyond the economic assumptions of an isolated optimizing individual (von Neumann & Morgenstern, 1949).

It has been noted (Ch. II) that the rational and individualistic utility maximizer of utilitarianism was rejected by Pareto, who, by introducing an element of 'machiavellianism' (Samuels, 1974) into his theory of social action, widened the scope of elements necessary for the explanation of such action. Also noted was the lack of this widened scope in the work of those applying welfare economics to political theory.

Game theory constitutes an attempt to remedy another utilitarian aspect of neo-classical economics, specifically the inadequate formulation of the situation of individual economic choice behaviour. The seminal work is the *Theory of games and economic behavior*, by J. von Neumann and O. Morgenstern.[54]

The 'Robinson Crusoe' Economy

Although economics is not a unified science a good deal of its theorizing takes off from that metaphor for individual economic behaviour which von Neumann and Morgenstern call the 'Robinson Crusoe economy' (p. 10).

In the Robinson Crusoe economy, the individual acts so as to maximixe his utility, subject to certain constraints

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[54] (1949) Although these virtually invented game theory, there was some earlier work in the same vein. (Borel, 1921).

from the environment. The outcomes depend only on this individual's choices, and do so in a known way.[55] In formal terms this is a maximum problem, varying in difficulty with the number of variables and the nature of the function to be maximized.

Criticism of this metaphor had traditionally been directed largely against the assumptions involved in the construct 'utility' and against the failure to consider 'social influences' on the actor which is inherent in this image of an isolated economic individual. Von Neumann and Morgenstern did not address themselves to the argument over utility,[56] and also characterize the majority of criticisms in terms of social influence as leaving the *formal* situation unchanged. One can 'handle' environmental influences (advertising, weather, etc.) which affect decisions by establishing probabilities for their occurrence, thereby leading to a mathematical expectation, which does not formally change the process of maximizing.

The 'Social Exchange' Economy

For them the major flaw in the metaphor is a lack of 'social sophistication' in another sense: the failure to consider in the model *more than one individual.* Such consideration is necessary, and with such consideration

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[55] It is choice under certainty.
[56] They did develop further the possibilities inherent in the ordinal model of utility as a basis for game theory - hence 'cardinal' utility, which is an assumption of most game theory.

important *formal* differences occur.[57]

In a 'social exchange' economy, an individual attempts to maximize in the usual way. But, to do this he must enter into exchange relations with others. And the results of his efforts will depend not just on his own actions but also on the actions of these others.

And these 'others' are not equivalent to environmental influences. For they are others *like himself* - each trying to maximize his own results, and guided by the same rational principles that he is.[58] For example, prices are not just part of the environment, they are created by the actions of other 'rational maximizers' (p. 11).

This means, then, that the individual must, in his rational behavior, take account of the rational behavior of others. This is a *formally* different problem, more complex than that in usual economic analysis.[59]

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[57] Although developed as a formalization of economic concepts, the development of game theory as a strictly mathematical system has been much more extensive than its conceptual applications in social science (including economics). The work of providing theorems, elaborating and adding to the number of formally distinct game types and of incorporating the formal work into the body of mathematics has taken place quite apart from any interpretation in the concepts of social science, contrary as this this may be to von Neumann and Morgenstern's approach (see 'Formalistic Functionalism', Ch. I). But this at least ensures a rich body of formal structures for conceptual interpretation.

[58] This makes the utilitarian idea of 'the greatest good for the greatest number' contradictory, say von Neumann and Morgenstern. For one cannot maximize two or more partly contrary functions at once.

[59] The Robinson Crusoe economic model is not wrong, but simply has much more limited application than it has been given (Bacharach, 1976). Game theory directly addresses the interdependence (with such problems as secrecy and collusion) and uncertainty which traditional analysis

[59]

Game theory is an attempt to formalize the essential aspects of this more complex problem. I shall now discuss these aspects of the 'game situation'.

C. The Game Situation

This section will include a discussion of the general relevance of the 'game situation' to politics. The issue is to what extent politics can be seen as a 'game'; that is, an attempt to find a rational approach to regulated conflict when the outcomes depend in part upon the actions of others.

Conflict and strategy

The initial formulation of the game situation is similar to that of social choice or classical economics: it is a choice or decision situation. One assumes a set of alternatives or choices; a number of individuals (1, 2 ... n); and a ranking of the alternatives. There are, associated with the choices, outcomes. Each of these outcomes is assumed to be a 'payoff'; that is, to have some 'utility' associated with it for each of the individuals.[60]

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[59](cont'd)ignored. However, when the number of individual becomes very large, the situation again approaches the Robinson Crusoe model, at least in theory, for one could predict 'averages', as with nature. In fact, the large numbers will usually combine into large coalitions and act like a small number of rational individuals (McKinsey, 1952: see Coalition Theory).

[60] Game theory typically assumes a greater than ordinal preference and utility ordering - called cardinal utility (Luce & Raiffa, 1957). Cardinal utility is derived from

[60]

However, there are crucial differences in the choice situation which are peculiar to the game situation and formalized in its theory.

The essential properties of the game situation are:

1. *conflict of interest*: people have different interests or desire the same scarce payoffs for themselves;
2. only *partial control of outcomes*: other 'players' are also able influence the outcomes as they attempt to maximize their payoffs;
3. *strategic thinking*: the other players are, like oneself, rational choosers and their rationality must be taken account of (by developing strategies).[61]

Politics as a Game

These three general aspects of human activity for which game theory provides a formalization are clearly relevant for politics, as several students of politics have noted. R. Axelrod considers that the 'essence' of politics is *conflict of interest* and evokes game theory as an especially

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[60](cont'd)ordinal preference either by further assuming that one can order his preferences as well as his alternatives, or by assuming that one can equate different probabilities of getting certain alternatives (op. cit. pp. 21-23).

[61] In terms of a theory of action, the game situation is one in which action necessarily becomes interaction. The relation between one's actions and outcomes is not determinate; rather, the relation between actions and outcomes are partly in the hands of others.

suitable vehicle for its study.[62] Also clearly, barring absolute dictatorship, there are a number individuals and groups which have some power to achieve favoured outcomes: for Aristotle, the definition of political rule is the problem of resolving conflict between such *different sources of power* in a society. In addition, much of political activity involves the *strategic interaction* of actors in order to achieve their aims. This aspect was left out of Arrow's social decision model; game theory specifically formalizes such activity. In particular the possibility of cooperation (collusion, coalition) to increase payoffs is evocative of political action (see Coalition Theory below).

Luce & Raiffa, in their *Games and decisions* (1957), compare the theorizing about politics and problems of policy formation done by 'moral philosophers' with the relative adequacy of game theory to embody real political rule. The usual model of the moral philosophers for resolving conflict is to assume 'true beliefs' (and complete knowledge) where one sees that interests are the same, or to assume 'fundamental values' which are shared, or, finally, to posit some absolute values or ideology.

But game theory can handle a problem when one does have complete knowledge and *still* disagrees. This is similar to the problems of politics: without assuming some absolute values, nor 'fundamental' aggreement, nor just lack of true

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[62] His book 'Conflict of interest' (1970) is a series of applications in specific institutional settings of the game theoretic framework.

knowledge; but rather real conflict and accurate knowledge of it, how does one reach a 'fair' solution?[63]

D. Classification of Games

Games are varied, and they can best be classified according to formal differences in their general characteristics. These formal differences may relate, in turn, to different political situations; that is, different game types can formally embody different political concepts.

I shall outline a number of formally distinct game types below: first I should like to set out some terms used in discussing games (Luce & Raiffa, 1957; Rapoport, 1966).

Game terms

The general game decision format was given above; the terms continue assuming this format.

1. Game: a set of rules or conventions for playing.
2. Choice: a particular alternative picked at some decision point.
3. Move: describes the situation at any decision point; what possible choices can be made. ('Black won by a clever choice on his 10th move').
4. Play: a particular sequence of choices until the game is terminated.
5. Outcome/payoff: at the end of the game each player has

[63] Some moral philosophers have developed their speculations from a game-theoretic perspective: Rawls has developed a 'theory of justice' based on the formal characteristics of 'Prisoners Dilemma'; Braithwaite has written 'Game Theory as a Tool For the Moral Philosopher (1969)'.

some outcome with an associated payoff.

6. Strategy: a 'policy' in which each player decides what to do in every choice situation, whatever other player(s) may do. Once each player has chosen a strategy, the course of the game is determined.[64]
7. Dominant strategy: a strategy at least at good (in outcomes) as any other, no matter what the other (rational) player does. Both or one or none of the players may have an dominant strategy, resulting in different 'equilibrium' outcomes or none. Finding such is the goal of much game theory.
8. 'Equilibrium' strategy: If both have a dominant strategy both will choose it. If one has the other will choose his best strategy assuming that the first will choose his dominant strategy: hence equilibrium again. If neither has a dominant strategy, it may still be possible to achieve equilibrium. Each player, assuming the rationality of the other, may choose a strategy such that he can ensure the best of the worst outcomes that the other player can give him. If both choose this 'best of the worst' they may achieve a equilibrium which is both a maximum and a minimum outcome: a minimax or

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[64] One may represent the strategies as a tree diagram in which every choice and outcome is shown (the extended form) or one may, with some loss of formal structure, 'normalize' the game by simply representing the different strategies as alternatives. The alternative strategies of each player can then be represented as rows of a matrix, with the determined outcomes as entries in the matrix (the 'normal' or rectangular form).

'saddlepoint' equilibrium.

Even in games without such a minimax equilibrium, it may be possible to apply rational considerations by using a mixed strategy: some probability mix of outcomes fixed according to the values of the outcomes.

Game Types

One may formally distinguish different varieties of games according to the following characteristics:

1. number of players: 2, small-N, large-N;
2. number of alternatives open to each player: 2, N (finite), infinite;
3. number of moves in one play of the game: 1, ... n;
4. information about choices open to the players: perfect, imperfect;
5. the total value of the outcome payments: constant sum (including zero-sum, where whatever is won by one player is lost by the other(s)), non-constant sum;
6. the nature of the players preferences with respect to the outcomes: strictly competitive, non-strictly competitive;
7. the existence of dominating or other equilibrium strategies/outcomes;
8. the possibility of communication and cooperation between players: cooperative, non-cooperative.

Some of these features imply others;[65] even so there is a great variety: within the class of '2X2' games (2 players and 2 alternatives or strategies), Rapoport and Guyer (1966) set out the possible combinations of constant and non-constant games, with different possible ordinal relations between outcomes, to arrive at a figure of 78 formally different 2X2 games (see also Harris, 1969).

E. Games and Political Situations

The relevance of the essential general features of games for some fundamental characteristics of politics was discussed earlier. Also, formally distinct game types were outlined.

I shall here comment on the relevance of particular game types to political situations, and discuss some elaborations of game theory developed in order to more adequately represent political behavior.

I shall state the argument for the special relevance of N-person game theory to the analysis of political situations (Riker, 1962), and the usefulness of conceiving political parties as coalition actors. It will furthermore be argued that, although some political contexts do not constitute game situations, decision-making bodies exhibit a tendency to evolve from a set of individual actors to a few disciplined party blocs; that is, toward a small-N coalition

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[65] For example, 'two-person-zero-sum' implies 'non-cooperative'.

situation (Penrose, 1946; Rae, 1969; see Ch. IV).

Formalizing political situations

R. Axelrod (1970) has outlined the necessary requirements for a formal (and empirically based) theory using the game theoretic approach.

There must be a specified *model of strategic interaction*: the participants, the range of strategies, the outcomes that result when all have selected strategies, and the payments. Many political stituations make this specification rather easy; for example, in a committee vote the participants would be the members (or coalitions), the strategies would be what one could do at a vote (vote for or against, abstain, bring an amendment), and so forth.

The model should allow for *varying* amounts of conflict of interest, in order to relate conflict to actual behaviour. Axelrod considers this an argument in favor of nonzero-sum games.

There must also be a *definition of conflict* of interest.

One must relate the conflict of interest to *behaviour*. The general hypothesis would be that 'conflict' will increase with higher conflict of interest, but what conflict *is* will vary.

One must then determine the possible, *outcomes*, get some measure of *utility* for them, and set up some method of

observing behaviour under conflict.

The above requirements apply to any game-theoretic formalization of a political situation: other specific formal features depend on the particular game type used.[66]

2X2 games and politics

2X2 games are those with 2 players and 2 alternatives or strategies, commonly set out (in the normalized form) as a 2X2 matrix with four outcomes. The class of 2X2 games has been the most developed formally, and many of the early political interpretations of game theory were based on this class. I shall briefly mention some of the variety of such games.

2-person zero-sum games

Most of the powerful theorems apply to this group (Luce & Raiffa, 1957). Zero-sum (or strictly competitive) games correspond to situations of complete conflict. One might question the applicability of such games, apart from parlour games and certain warlike situations, but Riker (1962) contends that the zero-sum aspect *is* politically important in that battle over control of policy can be formalized in this way - his (N-person) coalition theory is zero-sum (see below).

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[66] Axelrod applies in his analyses the 2X2 non zero-sum game 'Prisoner's Dilemma', and a 'bargaining game' of the same class, which he considers relate clearly to many political situations (see below). He also works with an N-person model.

2-person nonzero-sum cooperative games

These are arguably more relevant to political situations, if less clearly formalized. Here some sort of communication (or various types, including threats and coercion) is allowed. These games have been especially applied to situations where bargaining or arbitration occurs. Von Neumann and Morgenstern call their solution to the cooperative game - incorporating Pareto-optimality and minimum security levels among outcomes - the 'negotiation' set.

2-person nonzero-sum noncooperative games

In many political situations it is impossible to make binding commitments, yet there is not strict conflict. The most discussed game of this type is called Prisoner's Dilemma.[67] This game is unique among the 78 formally different 2X2 types, in that the outcomes are such that there is an equilibrium outcome in which both players are worse off than in another possible outcome. That is, if each chooses his 'best' alternative according to the dictates of individual (game-theoretic) rationality, the results for *both* are worse than if each was to do otherwise,

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[67] 'This game derives its name from the following story: Suppose two suspects are locked up in separate cells so that they cannot caommunicate. Let *a2* and *b2* denote confessing to a crime they are both accused of committing jointly, and let *a1* and *b1* denote not confessing. If neither confesses, both receive 2 years on some lesser charge. If one confesses but the other does not, the district attorney can convict both: but, as is typically the case, the one who confesses gets the lighter sentence - say 1 year - while the unrepentant suspect gets the maximum sentence - 20 years.' (Riker & Ordeshook, 1973: 223).

for example, to 'trust' the other.

This unique feature of prisoner's dilemma has given rise to a literature of political and social interpretation. The result can be seen as the 'breakdown' of the rational model. Of political (and economic) significance is that it is a *formal* example of conflict between individual interest and the 'interest of the whole: it is thus for some a refutation of Adam Smith and the tenet of 'methodological individualism' that the 'public interest' has no meaning apart from the aggregation of individual interests - there is, then , a 'general will' and a 'common good'.

N-person games and politics

On the face of it, N-person games would seem politically applicable on a number of different levels. In proportional electoral systems (Ch. V), the necessity for parties to negotiate to form a government offers scope for N-person analyses (the 'individuals' being parties), and several studies have done so. In pluralistic systems, majority governments are generally formed in the election itself, so coalitions of parties would seem unnecessary. However, minority situations would lead to some pressure for coalition behavior. On the larger scale, one could consider a large-N analysis of voter coalitions: this is formally a very different analysis from small-N situations.

One important series of articles suggests that, even in situations with large numbers of participants, small-N theory could become appropriate, because political systems and decision- making bodies tend to *evolve* toward the small-N person (party, faction, etc.) situation. Thus N-party coalitions could be part of an evolutionary analysis of the development of factions and parties in such situations (see Ch. IV).

N-person games and political coalitions

W. Riker (1962) has loosely characterized the types of problems and the different ways of 'solving' those problems, in games with different numbers of players.

In a 'game' of one person, one attempts to maximize one's gains versus some range of chance in nature by choosing some technique of maximization.

In a 2-person game one has the problem of conflict. One attempts to get 'the best of the worst' by selecting a strategy.

In games of 3 or more one has, in addition to conflict, the possibilities of certain *parallel interests* and resulting *coalition or collusion*. One selects partners, and then strategies.

There is in fact an important logical difference between 2-person and N-person games. The possibility of players cooperating with each other, of forming coalitions, gives N-person game theory a quite different, less settled, status than 2-person game theory, where some game types are

essentially solved. In particular, there are problems with formalizing collusion and coalition.

Yet it is probably N-person game theory which is of greatest relevance for political systems. Riker argues that this is so.

Riker (1962) begins by asking whether there is some *distinctively political* kind of action. He argues that there is, and, following Easton, defines the function of the political system or of political action as 'the authoritative allocation of value'. The word 'authoritative' signifies that politics is a matter of government, the word 'value' signifies that politics is about the public morality or good, and 'allocation' signifies that politics is about the public *struggle for power*.

Now, if allocation refers to some social process of deciding how some action shall be carried out, then the subject of Political Science is *decision-making* (not all decisions: those concerning value which are assumed to be authoritative).

Decisions are of a number of sorts; by individuals or by groups; by conscious process or by some mechanical device. 'Mechanical' decisions by groups are such as the market would make. But if a decision is a conscious process by groups of (more than two) persons, the process of making decisions is always the same: it is a process of *forming coalitions*.

So, for Riker, politics is decision-making, done through the formation of coalitions. And, the key to the study of politics is a general model of the formation of coalitions; that is, the theory of N-person games. The application of N-person game theory to politics was given a signal start with the publication in 1962 of his book, '*The theory of political coalitions*'. Since then much work has been done in political 'coalition theory'.

F. Coalition Theory

This constitutes the largest area of formal political theory in a game-theoretic framework. The basic problem in such work is to determine which coalition will form out of a number of possibilities. The trend in the literature seems to be to move from the sole consideration of 'weights' or power as coalition determinants toward the additional consideration of policy (issues, ideology).

The following versions of regions of coalition formation have been adduced:

Power based coalitions

1. 'The core' of undominated coalitions (von Newmann & Morgenstern, 1949).
2. 'The stable set' containing any minimum winning coalition (von Neumann & Morgenstern, 1949).
3. 'The bargaining set' of the minimum winning coalition with the smallest number of parties (Leiserson, 1970a).
4. 'The size principle' specifying the minimum winning

coalition with the smallest weight (Riker, 1962).

5. 'The information effect' specifying deviations from the size principle with imperfect information (Riker, 1962).

Coalitions with policy dimensions

6. 'Psi stability', the recognition that there are social considerations determining which coalitions form and remain stable (Luce & Raiffa, 1957).
7. 'Minimum *connected* winning coalitions' the connection being graph theoretically embodied issue-relatedness (Axelrod, 1970).
8. 'Minimum winning coalitions with lowest ideological diversity' (Leiserson, 1970c).
9. 'Coalitions as policy distance minimization' (DeSwann, 1970).
10. 'The size principle and connected subgraphs as coalition determinants' (Rosenthal, 1968).

Political coalitions

Like 2X2 game theory, the assumption in N-person theory is that individuals try to maximize their payments. As mentioned, in N-person theory this takes the form of coalition formation. The aim of the game theorist is to construct some formulation of 'equilibrium' social behaviour and study its working out. In N-person game theory, this is the attempt to find stable coalitions. Before discussing the above-mentioned attempts, I shall set out some terms for N-person theory.

Coalition terms

1. Decision: a selection among alternatives.
2. Payoff: reward to participants in a decision.
3. Weight: resources of a participant. Some critical quantity of weights is necessary to control a decision.
4. Social unit: participants (party, voter, bloc) which follow some strategy.
5. Coalition: joint use of resources/weights. A number of social units forming one unit.
6. Coalition structure: a partition of the N players. There are many possible such for even a small-N game (2-to-the-Nth power).
7. Winning coalition: a coalition with sufficient weight to control the decision. One half of the possible coalitions will be winning.

The coalition situation

The 'coalition situation' then, is that there is a decision to be made, with N participants, each trying to maximize his share of the value (payoff). No single alternative maximizes the payoff to all (there is conflict). No participant has sufficient weight to control the decision alone. Therefore, there is some common interest in achieving the decision. Such a situation sets the stage for the formation of coalitions.

As mentioned, there are a large number of possible coalitions. But, in fact, there will be found restraints limiting coalitions which may form. Game theory attempts to

specify such restraints in the formal system in the hope that they can then be found in 'reality'.

Power Based Coalitions

1. *'The core' of undominated coalitions*

Characteristic functions

To describe the possibilities of coalitions von Neumann and Morgenstern first formalized the idea of coalitions in set theory (as partitions) and then developed the notion of a 'characteristic function' as a measure on the sets. This idea was then used to specify some limit on coalition formation. They defined the 'value' of a coalition as what it could assure itself of if everyone were against it; that is, if a complementary coalition formed. This is the typicial 'conservative' approach; the value is a minimum security level. The characteristic function then was a function 'v' with the following properties: v(O) = O. and v(S union T) is greater or equal to v(S) + v(T).

Von Neumann and Morgenstern discuss a 3-person game called 'Couples' that illustrates the characteristic function (p. 222). In this game each player makes a choice alone of one of the other members of the game. If in this choice two players choose each other they are a couple, and receive a payment of 1/2 unit each, while the third loses 1 unit. If no couple forms, all payments are 0. For this game, if the coalition S has 0, 1, 2, or 3 members, the

characteristic function v(S) equals 0, -1, 1, or 0 units respectively.

There exist other versions of this game, including the five-person 'Talleyrand', and the constant sum version 'Division'. According to Riker, these games are elementary models of the essence of politics - which is choosing sides as parties, factions, coalitions, alliances etc.

Imputations

The characteristic function shows strategic possibilities: what a coalition can get if it does form. But it does not say much about what coalitions *will* form, or anything about the payoffs. An attack in terms of payoffs is the idea of 'imputations'. Imputations are defined as coalitions which for the individuals forming them are at least as good as they would get alone and for the group offer a payment sufficient that no one could get more without hurting someone else (are 'Pareto' optimal).

Are there 'stable' imputations? Von Neumann and Morgenstern suggest that they will be found in a 'core' of undominated coalitions: the set of coalitions at least as good (in payoffs) as any others possible in the game.

But often there is no core.

These concepts and others offer formalized approaches to coalition solutions in terms of payoffs. But one might address oneself directly to the 'weights' or power of the coalitions and ask questions about the 'size' of coalitions that may form.

2. *The 'stable set' of minimum winning coalitions.*

Von Neumann and Morgenstern offer one such solution. They predict (1949, p. 420) that no coalition will include parties that are not needed to win: that is, any minimum winning coalition will be stable. The reasoning is that in any zero-sum game the winners will not want to share the payoffs with any more 'parties' than necessary.

3. *The 'bargaining set' of minimum winning coalitions with fewest parties.*

Lieserson develops his proposition in the context of a study (1970a) of coalition government in Japan. One party (LDP) has ruled Japan for some time, and the coalitions are among factions of this one party. The coalition considerations are long-term ones, with the leader entering into agreements with present supporters and previous opponents with cabinet posts as the payoffs.

In arriving at his prediction of which coalitions will form, Lieserson accepts that it will be nesessarily a winning coalition, with no parties not needed to win (a minimal coalition). He also considers that every winning coalition will be equally valuable in payoffs. For him the criterion of stability among minimum winning coalitions will be the process of bargaining. And this will indicate as few parties as possible. Coalitions will tend to form because the bargaining process over the formation and mantenance of the coalition will be easier if the coalition has the fewest 'factions' as members.

4. *'The size principle': the minimum winning coalition with smallest weight.*

It will be recalled that W. Riker stressed the special relevance of N-person game theory to politics. In his '*Theory of political coalitions*' (1962) and in subsequent work (1973), he advances his 'size principle' as the key to formalizing coalition formation and stability.

Riker uses the idea of characteristic function and some assumptions about individual rationality to give initial limits to the value range of possible coalitions. Then, from this framework, he adds 'political' restrictions. These are: only winning coalitions have positive value; the primary goal of players is to form winning coalitions; winning coalitions are associated with imputations in which all members receive positive payoffs; and, members of a coalition can increase the size of the coalition whenever they wish.

From these further restrictions on the range of the characteristic functions, Riker derives the size principle: 'In social situations similar to N-person zero-sum games with side payments, participants create coalitions just as large as they believe will ensure winning and no larger'(1962, p. 32).

Riker goes on to discuss a great many political situations to which his size principle might be applied; in particular situations where overwhelming majorities (including 'grand coalitions') are found. He argues that

they tend, in a variety of ways, to become smaller.[68]

Riker also develops a 'dynamic' model of coalition formation, including discussion of side payments, proto-coalitions, and stage-by-stage strategies; and discusses the problem of stability in *just* minimum winning coalitions.[69]

5. *The 'information effect': deviations from the size principle.*

Riker's first model posits perfect information in ensuring minimum winning coalitions. If information is not perfect, coalitions must enlarge themselves above minimum winning size to increase the liklihood of winning.

Coalitions With Policy Dimensions

6. *'Phi-stability': social conditions for coalition formation and stability*

Riker's work focusses mainly on power or weights (with payoffs). He does discuss ideology: for example, he considers Luce & Raiffa's concept of 'phi-stability' (1957), which posits specific sociological conditions that might act as 'boundary conditions', restricting particular *changes* in coalitions. Riker offers as an example of this the practice in Italy of excluding communists from all coalitions. But

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[68] For a similar conclusion, differently formalized, see 'The rise of third parties', Ch. IV.

[69] His discussion of stability includes a game-theoretic derivation of 'Duverger's Law' (the tendancy to 2-party systems: Ch. IV).

he does not regard such situations to be general restrictions on coalition structure.

Nevertheless, there has been a turn in coalition theory to a more systematic consideration of policy and ideology.[70] I should like now to consider some work in this vein.

7. *Minimum connected winning coalitions*

R. Axelrod has undertaken a game-theoretic analysis of 'conflict of interest' as a political problem in a variety of situations with varying 'strategic structures' (bureaucracies, parliamentary committees, etc.). More to the point here, he examines conflict of interest in 'Multiparty Coalitions in Parliamentary Democracies', with an application to the Italian parliament.

His concern is with proportional systems (see Ch. IV), in which the election of party representatives is not ordinarily sufficient to give any one party control of the legislative assembly (and, consequently, public policy).

In other words, elections do not result in majority governments, as they typically do in pluralistic systems. It requires a further process of forming coalitions *between* parties in order to form a group large enough to control the legislature.

It is this process which Axelrod examines. He considers parties as 'individuals' capable of entering into coalitions, and uses N-person game theory to estimate which

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[70] Perhaps more general ideological variables would provide 'general' restrictions.

coalitions are likely to form and remain stable.

Axelrod emphasises that bald prediction would be difficult; for example, in an 8 party system, there would be 256 different possible coalitions, including 128 which could govern. His search is for stable solutions; clearly not all combinations would be equally likely. The key for him is that those coalitions are most likely to form which have the *least conflict of interest*. The conflict of interest approach does not require a clear knowledge of policy differences; low conflict of interest coalitions will simply be easier to negotiate and will in practice last longer.

Party Policy Space

Axelrod next 'formalizes' his argument. Developing a 'spatial model' (see Ch. IV), Axelrod uses an *ordinal policy space* to represent formally the positions of the political parties along a left-to-right ideological dimension. Only an ordinal arrangement need be assumed because one can test the theory without assuming intervals between the parties to be any particular size. Nor need utility be linear with distance: there must only be assumed what Black has called 'single - peaked' preference (see Ch. II); that is, that of any two party positions on the same side of a party, the party should prefer the nearer.

Conflict of Interest Defined

Conflict of interest can be now measured, at least relatively, using the ordinal policy space. A party or coalition will have more conflict of interest with a party

twice removed from it than with an adjacent party. It follows that 'dispersion' of a coalition can give a relative measure of conflict of interest.

For example, assume seven parties ordered from left to right: A B C D E F G. One cannot compare (A B C) with (B C D). But one *can* be certain that the coalition (A B C) is less 'dispersed', hence has less conflict of interest, than coalition (A B D) - in graph theoretic terms a 'connected' coalition will have the least conflict for its size. Also, it follows that the smaller the size the less conflict; that is, coalition (A B C D) will have less conflict than (A B C).

Coalition Structure Predicted

Axelrod then sets out the structure of coalition in game-theoretic terms: there are N parties; policy can be determined only by a group of parties controlling enough seats to give and maintain a vote of confidence in the chosen cabinet; the less conflict of interest in any coalition the more likely it is to form and remain stable; the possible outcomes are each of the possible combinations of parties into coalitions; the parties utilities are sufficiently specified by their positions in the policy space.

The game theoretic framework leads Axelrod to predict three properties of 'equilibrium' coalitions - those which tend to form and remain stable. One is clearly power related and the others are policy related.

1. The coalition must be a 'winning coalition'.
2. The coalition should be 'connected', to reduce conflict of interest.
3. The coalition should be as small as possible, to reduce conflict.

In summary, the predicted coalitions will be minimum connected winning coalitions.

This radically restricts predicted coalitions. With seven parties, there are 128 possible coalitions, 64 of which are winning. Of these, 28 are connected - consist of adjacent members. And of these, the minimal such are four: ABCD, BCDE, CDEF, and DEFG.

Axelrod then applies his model to actual coalitions formed in Italy over an 18 year period. In doing this he compares his predictions and results with four of the zero-sum coalition models mentioned above (von Neumann & Morgenstern, 1949; Leiserson, 1970a; Riker, 1962-both versions). Of the 17 coalition governments which existed during that period, the four models predicted 3, 1, 0, and 5 respectively. Axelrod's predicted 10. He ascribes this partly to the non-zero sum nature of his model.

8. *Minimum ideological diversity in minimum winning coalitions.*

An approach very similar to Axelrod's is Lieserson's (1970c) model of coalition formation in terms of 'ideological diversity'. This model assumes that political parties search for other parties that are close to them

ideologically; closeness being defined spatially. Thus coalitions formed will have minimum diversity.

Others who have developed coalition theories which include ideological considerations as important formal elements in determining stable formations are DeSwann (1970) and Rosenthal (1968).[71]

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[71] Taking off from the work of Riker and Lieserson, Rosenthal is attempting to develop models for the *simulation* of coalition formation in the environment of other political processes (resource allocation, voting, policies).

G. Game Theory and Functionalism

In all games the characteristic problem is the attempt to find some extremum or equilibrium result (dominance, strict dominance, etc.) from the 'rational' action of the participants. These are 'solutions' to the games; and they are its 'functional' nature.

From the foregoing discussion, it appears that important aspects of politics can be understood as operating to attain the 'equilibrium' of a *minimum winning coalition*. This is an 'essential' variable of power (weights, voting strength) but also interacts with policy (ideology, preferences).

Further support for the political importance of the minimum winning coalition will be seen in the discussion of the evolution of decision-making bodies.

Static nature of game theory

The problem often noted is that game theory is in mathematical terms 'static'. What are the possibilities of a 'dynamic' game theory?

One possibility in the political situation is the work done on repeated games, which might offer some chance of achieving an optimal path. In dynamic programming the equivalent to the optimal path is a series of discrete optimal policies: segments where one makes repeated decisions as to maximization within a short-term framework.

The phenomena of repeated voting in decision bodies, and of repeated elections could be seen as at least

suggestive of an analysis in dynamic terms (see Ch.IV, the 'evolution' into coalition stituations).

And there have been recent analyses of politics in dynamic terms (Schofield, 1978).

Individualistic nature of game theory

Is game theory only relevant at the individual level? Axelrod's approach could be characterized as an aggregating method, taking game theory beyond the individual level.

And N-person game theory could be termed a 'pseudo-aggregating' approach. One can conceive N-person game theory as the search for an essential variable with the formal character of an extremum. I think that the 'minimum winning coalition' would be one such, and it would not only be individually rational but socially functional - an aggregate extremum obtained by actors in a particular set of social institutions (Ch. V).

IV. Other Formalized Areas of Political Science

A. Introduction

Apart from the literatures of social choice and game theory there is a political science literature dealing with a number of more specific and variously formalized relations. I shall briefly discuss some that are of interest, in particular a geometric interpretation of electoral competition, a number of articles dealing with the evolution of dual 'blocs' in decision-making bodies (with special relevance for coalition theory), and some approaches to the problem of the 'vote/seat relation' (which complement social choice theory).

B. Spatial Models of Party Competition

In such 'geometric' models (Downs, 1957; Stokes, 1963; Axelrod, 1970; Riker & Ordeshook, 1973) individual voter preferences are summed on one or more spatially represented 'policy dimensions'; the populace is viewed as a distribution of points in the space produced by these dimensions. Parties or candidates compete within the policy space to maximize their liklihood of winning by arranging their policies (or the representation of them) in least variance with the policy distribution of the population.

Some specific political system features considered in these models are: the apportionment of policy conflict among different levels of the system, the effect of crosscutting

policy spaces (Dahl, 1956), and the interaction of party policies and the number of parties (e.g. two parties and centrist policies, many parties and divisive policies).

'Spatial analysis' has its origin in the 'location theory' of H. Hotelling (1929), who used the idea of a one-dimensional market to explain why competing firms are generally found close together (Stokes, 1963). Firms will, he said, cluster together at the centre of the market in order to minimize 'transportation costs' for consumers. Hotelling suggested that this could also be applied to politics: by substituting voters for consumers, parties for firms, and the costs of ideological 'distance' for transportation costs, one could explain the often noted similarity of Republican and Democratic party platforms to each other (on a liberal-conservative dimension).

A. Downs, in 'An economic theory of democracy' (1957), adapted Hotelling's work to demonstrate that, by analysis of the distribution of *voters* along an ideological dimension, one could explain such political phenomena as the strategic choices of parties, the emergence of new parties, and the number of parties. Unimodal, bimodal and multimodal distributions are found to lead to different 'equilibrium' party systems: 2 'adjacent' parties, 2 distinctive parties, and multi-party systems, respectively.

With variously altered formal and empirical

assumptions,[72] this mode of analysis has continued to the present.[73]

'Spatial' Conflict of interest in society

In his book 'Conflict of interest' (1970), R. Axelrod addresses himself to what he considers has been a fundamental theme in Western political thought since Aristotle: 'the relationship between the common interests and the divergent interests of a political community' (p. 144).

Axelrod approaches this 'dilemma of collective action' by developing a formalization of the concept of 'conflict of interest', and using it to analyse a series of different political situations (see Ch. III). His contention is that the idea of 'conflict of interest' addresses the fundamental aspect of politics, and that his is a general 'theory of divergent goals' in the context of politics.

In the course of his analyses, he addresses himself to conflict in a total society, in which task he makes use of a spatial model of society. He adjusts his formalization of conflict of interest to fit it for the representation of conflict among large numbers in this spatial model, and then applies his model to the problem of 'how apportionment (of conflict of interest) should be performed in representative democracies', and to the treatment of the 'pluralist' theory

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[72] For a review and critique, see Ordeshook, Reprint No. 180.
[73] For a recent discussion in the context of the 1980 U.S. Presidential election, see Steen, 1980).

of crosscutting and overlapping ideological cleavages (Dahl, 1956).

C. Power and Policy in the Evolution of Decision-making Bodies

The literature includes a number of arguments with respect to the possibilities of policy success and voting power under varying decision rules and varying degrees of cooperation among voters (Penrose, 1946; Kemeny, 1959; Rae, 1969; Axelrod, 1970). Formal considerations in these relations offer support for particular decision rules (e.g. bare majority) and imply strategic advantages to the formation of voting blocs and parties.

Political manicheism

M. Duverger, in his analysis of political parties (1951), discusses that evolution to 'dualism' which he regards as an essential characteristic of political life.

According to Duverger, public opinion exhibits a 'deep-seated tendancy to divide into two major rival factions'. This 'natural political dualism' is the political version of a 'deep-seated Manicheism' to be encountered in much sociological work: radical and conservative temperaments, bourgeoisie and proletariat, and so forth. Indeed, all the great factional conflicts throughout history exhibit such a dualism (p. 216).

This is the nature of political choice and conflict. Every policy involves a choice between two kinds of solution

- action requires such a choice. So the centre does not exist in politics.

The development of two-party systems

For Duverger, the natural movement of societies is toward the two-party system. To be adequate to the deep-seated Manicheism of political choice and action, party systems should reflect clearly the basic cleavage which develops.[74] Duverger's position clearly supports the earlier contention that coalition theory is adequate to some essential aspect of politics; indeed, small-N coalition theory would seem to formally embody a developmental goal of political life. The articles below offer *formal* support for this evolution to 'dualism' in political systems.

The elementary statistics of majority voting

In the literature discussed in chapter III, the small-N coalition situation assumed the extreme 'conservative' situation of complementary opposing coalitions. The opposite extreme can be found in an article by L. Penrose, 'The elementary statistics of majority voting' (1946). Penrose sets forth as a basis for discussion a situation in which isolated individuals vote in a random manner issue by issue in a decision-making body.

He then describes and formalizes the tremendous advantage to be gained in this situation if even a small proportion of these individuals form a *bloc*; that is, agree

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[74] 'Duverger's Law' is a widely supported statement of the tendancy to the two-party extremum in pluralist systems (see below).

in advance to vote the same way on any decision.[75]

Having demonstrated the principle, Penrose goes on to discuss the even more dramatic effects of such blocs when a number of decision-making bodies are placed in a heirarchical relationship, and to recommend for deliberative bodies such as the U.N. alterations in voting strength which take account of the effect of blocs.

The power of bloc voting

The advantage for blocs constitutes for Penrose an increase in voting power. The increase depends, as he points out, on the other voters continuing to vote in a random manner; any contrary bloc would reduce the voting power just as dramatically as the original bloc increased it. And, indeed, such continued random voting is very unlikely. Nor is this accidental; it is to be expected from the very advantage that Penrose demonstrates.[76]

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[75] In a voting population of N members, a bloc of 'N to the square root' will be able to control decisions about 85 percent of the time. For example, if the Alberta population were to vote on issues as referenda, given a voting population of one million and the above conditions, a bloc of just one thousand people could exert such control.

[76] In a relatively small body (of parliamentary size) this advantage would quickly become evident, even supposing no evident cooperation and secret ballotting. The fact that issues are connected with each other would result in the *evident* passage of a policy over some time, and certain groups would show less than expected success; this would occur unless the vote of the bloc itself were to be completely divorced form policy considerations, or if the existence of a policy could be hidden.

This could occur in local politics when the bloc acts to influence policies which are not clearly evident. In fact, much of 'local politics' might be viewed in terms of the formation and maintenance of 'resolute' blocs.

The significance of Penrose is that he sets out a situation at polar extremes from the 'coalition situation'. Then, he shows the formation of a 'coalition' will be highly advantageous. Looked at another way, this result throws light on the evolutionary[77] development toward party systems, cabinet rule, and so forth.

The 'best' decision rule

Another article applying statistical approaches to the operation and evolution of decision-making bodies is Douglas Rae's 'Decision rules and individual values in constitutional choice' (1969).

Rae's problem is the choice of a decision-rule in a 'political committee'. By political committee he means a decision-making body, by decision-rule he means a rule of procedure specifying the number of votes necessary for the passage of some issue.

Decision rules as values

He sets out a range of such rules: in a committee of N members, the number necessary for passage can vary from 1 ('individual initiative'), through majority rule, to N ('unanimity'). He wants to choose some 'best' rule from

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[77] Indeed, the situation is 'evolutionary' in a classical sense. In a voting population of the above type, small differences in policy distortion give an advantage to some which would in turn increase the liklihood of the differences. Thus, even supposing an initially random vote, the situation is unstable: even in a relatively large population with poor communication there would be a sort of 'natural selection' of blocs, the situation moving toward the ultimate 'stability' of a minimum winning coalition (with complementary opposition).

this set of N possibilities.

His procedure is to specify a normative criterion of individual success and then to see how each possible decision rule measures up on this criterion, according to a statistical model of 'expected frequency of success' for *all* individuals in the committee. That is, he assumes individuals seeking maximmum success; his rule is, however, a measure of social success - a minimum necessary advantage for all.

Rae ends up with majority rule as the best, which is not surprising. But in the course of his argument he formalizes a number of concepts which have considerable import for political theory; more to the point here are several features of his argument which support the contention of a tendancy in decision-making bodies towards the formation of a 'small-N coalition' situation.

The development of factions

Rae's individual assumptions are much as were Penrose's: isolated individuals, aiming for a maximum of gain. He then discusses departures from this, of which the development of factions is the most interesting. In particular, as the decision system evolves, there develops, says Rae, 'an incentive for the formation of a minimum winning coalition'.[78]

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[78] Rae's analysis is in terms of power, here a 'passed' policy. He shows the the importance of the development toward the 'coalition situation' (or what Duverger calls the 'dichotomy of power') not in explicit statement, but in the very high slope (of his 'success' function) as the decision

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Structural balance in social groups

Relevant for present concerns is the formalization (Berger, 1962) of Heider's theory of 'structural balance' in social groups. In a group, there are typically certain positive and negative relations which obtain among members such that the total situation is regarded as pleasant or unpleasant. It is further assumed that members will try to change unpleasant to pleasant situations (and to remain in pleasant) by altering their relations in a variety of ways: hence the unpleasant situations are 'unbalanced' and the pleasant ones are balanced.

Heider's work on this problem was analysed by Cartwright & Harary (1956). Using signed symmetric graphs (positive & negative) they developed a graph-theoretic formalization of the concept of structural balance.

Balanced graphs and two-party structure

With this formal definition of balance they used the formal systems of graph theory to derive a theorem with considerable interest for politics. This is the 'structure theorem for signed graphs' in which it is shown that a graph is balanced if and only if it is possible to divide the points into two sets, such that all positive connections occur between points in the same set and all negative connections occur between points in the other set.

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[78](cont'd)rule N/2 is approached.

Interpreted politically, the theorem shows, according to Kemeny (1957), that a political body is balanced if and only if it is possible to impose a two-party structure on it. That is, the structural balance theorem is a graph theoretical formalization of the winning coalition/losing coalition situation, of the game theoretic extreme. Once put in this form, it was possible to define degrees of balance, balance at a point, and local balance (proto-coalitions). It remains to be seen if a *tendancy* to approach this situation has been proven in graph theoretic terms.

The evolution of political parties

In chapter III, I stated the argument for the special relevance of N-person game theory to the analysis of political situations, and the usefulness of conceiving political parties as coalition actors.

The import of the above articles is that decision-making bodies do exhibit a tendancy to evolve from a set of individual actors into a few well-disciplined party-blocs; that is, toward a small-N coalition situation.[79]

This tendancy could probably be supported empirically through an examination of the early development of those political systems which are now conceivable as small-N systems. The British system, in its long-term evolution

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[79] Indeed, toward the 'conservative' set/complement situation often assumed in coalition theory.

from occasional caucuses and electoral groupings, through organized factions and permanent branches, to disciplined parties and extra-parliamentary interest groups could be so studied.

One might examine the emergence of coalitions in those systems which have been arbitrarily set up in recent times, for example, Germany and Japan.[80]

However useful such empirical evidence might be, more to the point in this study is the literature which *formally* demonstrates that one should, on theoretical grounds, expect a transition to small-N coalitions. I think that the above literature, examining formal features of decision-making bodies, does offer 'explanatory' support for the existence of small-N coalition situations in developed political systems.[81]

In addition, the literature offers formal structures to use in any theoretical examination of the evolution of political systems.

The above literature also offers the possibility of a 'dynamic' functional formulation, in terms of essential variables for power and policy. Some formal features are indicated: the essential variable 'policy' (as maximization of policy success) would lead to the formation of

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[80] Germany moved quickly from a myriad of minor special interest parties after the war to the present situation of two 'almost winning' parties and a third minor centrist party which has formed 'just winning' coalitions with each of the others (Merkl, 1970).

[81] Riker's model itself contains this 'tendancy' (1962, p. 182). Riker mentions it as supporting 'Duverger's Law'.

increasingly 'resolute' blocs until the attainment of the 'minimum winning coalition'. There would be a power 'plateau' in the minimum winning coalition, with no power for losing coalitions - hence the often-spoken of 'dichotomy of power'. And, since all minimum winning coalitions have the same power, further specification of equilibrium coalitions would be in policy terms (Lieserson, 1970).

D. The Vote/Seat Relation in Electoral Systems

In the aggregation of votes to parliamentary seats, some political systems render the proportions equal while other systems exhibit a non-linear relation between vote/seat proportions. Several approaches to this 'representation function' (Spafford, 1975) have been taken: verbal, methodological or curve fitting, probabilistic, axiomatic, and a 'political thermodynamic' interpretation (Duverger, 1951/59; Kendall & Stuart, 1950; March, 1957; Rae, 1967; Qualter, 1968; Theil, 1969).

This difference in electoral systems has also been the point of departure for relations to other varying properties, such as the number of parties ('The Wasted Vote Thesis', Fisher, 1973) and the nature of party competition (Pinard, 1971; Spafford, 1972).

Proportional and plurality electoral systems

In a typical European election there are a variety of mechanisms, some very complex, the purpose of which are to ensure that the proportion of seats in the elective assembly

will be approximately the same for each party as the proportion of votes cast for that party. This 'proportionality', from which such systems get their most common name, is not exact: the mechanisms work out with some error in assigning seats to members.[82]

The proportional system, now the most common, shows marked contrasts to the plurality or 'British' systems.[83] There are differences in many electoral laws and practices, but the most important difference in *result* is that, in the British system, the lack of any such mechanisms means that the relation between votes cast and seats achieved is very erratic. There are, however, some relatively stable characteristics in the long term (see below).

The work I shall now discuss attempts to *specify the relations* between votes and seats in non-proportional systems in formal terms.[84]

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[82] In social decision terms the system imposes an 'axiom of equality' on the seat/vote relation: all votes count equally in parliamentary representation. However, there are some provisions in constitutions which introduce distortion; for example, the West German constitutional provision barring representation entirely to very small parties.

[83] The 'British' system is older, but is now confined to countries whose electoral systems have derived from the British (Canada, Australia, New Zealand). Proportional systems developed in association with the rise of socialist parties in Europe; being either adopted by countries first established during this period (Italy, Germany), or adopted then through constitutional change (Sweden, Switzerland, Norway).

[84] These can be seen as complementary to the social choice approach: here are attempts to formalize, starting from real political sysems, rather than approaching politics from the formal systems.

'Duverger's Law': two-party and multi-party systems

In his discussion of the 'Manicheism' of politicial life M. Duverger has occasion to discuss those features of political systems which both enhance and inhibit the 'natural' tendancy to duality. In particular, he offers an explanation for the clearly superior progress of 'Anglo-saxon countries' toward that natural state. His explanation is in terms of the 'electoral regime' that such countries use. He states the cause and effect as follows: 'the simple-majority single-ballot system (in plurality systems) favours the two-party system' (p. 217). Duverger considers this a 'true sociological law', and goes on to examine the process in several countries; in particular noting how the two-party system is restored through the underrepresentation of third parties (and their eventual elimination).

The 'Cube Law' in electoral systems

Since the rise of European proportional electoral systems there has been a lively argument in Britain over the desireability of changing the British electoral system into a proportional one (Hermans, 1941). The proportional systems seem more democratic: the wildly varying relation between votes and seats in British systems makes election results seem haphazard. Yet, others have pointed to the instability of European systems and to the 'degeneracy' of their party and cabinet systems. The two-party system was considered to be necessary for good government.

So, interest in the vote/seat function was high, and with the 'rediscovery' of an early formal characterization of it, a good deal of consideration was given to what has become known as the 'cube law'.

The 'cube law' states that, if one were to plot party results in terms of vote and seat proportions on a graph, the line most able to sumarize the results would be an 'S' curve, such as is generated by a cubic function. Compared with the 'straight line' relation of proportional systems, the cubic law shows a lower seat/vote ratio for parties receiving less than one-half the votes, and higher ratio for those receiving more; that is, small parties are underrepresented; large parties are overrepresented. Also, the high slope of the cubic function near the point of inflection (where underrepresentation becomes overrepresentation) is indicative of a highly volatile relation between seats and votes: small changes in voting strength can have great effects on seat strength. In addition, one gets the likelihood of 'artificial majorities': especially where there are more than two parties, the largest party will often get a majority of seats in spite of not getting a majority of votes.

Studies have analysed statistical requirements for the cube law to operate, attempted to extend it two more than two parties, considered the rise and fall (see below) of 'overwhelming majorities', examined other influences, and in general attempted to explain the factors entering into the

determination of the 'representation' function.

The rise of third parties

The actual relation between vote and seat proportions varies not only with statistical distributions in the population, but with the number of choices on the ballot; that is, with the number of parties in competition for the vote/seat proportions.

The presence of three parties in the relation greatly complicates the estimation of the function, and few clear results have been obtained. One exception to this has been a special situation in a multi-party system when one of the parties does very well (gets a huge majority) and the second party does very poorly. There then occurs the phenomenon of the rapid rise of a third alternative which increases its voting strength very markedly. This does not mean a stable multi-party system as a result: it is more in the nature of an evolutionary development in the history of two-party systems. M. Pinard, in 'The rise of a third party' (1971), analizes election results involving the Social Credit party and shows this effect. D. Spafford (1975) approaches the relation with a different formalization, demonstrating a partial derivative with the appropriate slope in the same electoral situation, thus confirming Pinard's curve-fitting results.[85]

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[85] This seems to demonstrate for plurality or non-proportional systems the same formal constraint on the size of a governing party as the predication of *minimum* winning coalitions does for political coalitions.

Political entropy in elections

H. Theil, in 'The desired political entropy', addresses himself to a call in the European context for a vote/seat relationship which deviates from proportionality *toward* the cube-law. His call is for a 'square' law; that is, for a squaring of the voting probabilities to achieve *seat* proportions which exaggerate the support of larger parties and diminish the support of smaller parties. This would, he avers, cut down on the long months of coalition negotiation after elections in order to form governments.

Theil first offers a general formalization of the 'family of representation systems' in terms of the 'power' of the voting proportions which give the seat proportions. He next states what he considers the real nature of the criticisms of representation functions: critics are unhappy with the degree of dividedness in the total representation; it is either regarded as too large or too small.

Theil then offers 'entropy' as a natural measure of dividedness to use in this context and provides a recommendation of his own: its import is that the electorate ought to be surprized as little as possible. Whatever the sense of his recommendation,[86] Theil offers a clear formal

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[86] It might be considered that a *high* level of surprise would be better. The great uncertainty of result and the quick resolution on election night ('The Canadian people have spoken ...') in non-proportional systems contrast with the minute uncertainties regarding seat proportions (since voting proportions are known from polls) and the drawn out *resolution* of the result which often obtain in proportional systems.

characterization of the range of seat/vote relations (the powers) and a criterion for evaluating them (entropy) which admits of functional interpretation.

E. Summary

Besides social choice and game theory there are in political science many attempts to formalize important political concepts. In the above discussion I have selected some that are of particular interest, in that they offer alternative formalizations to the work in social choice and game theory or handle formal structures and concepts in a way which appear to offer possible use in a functional theory.

I briefly noted the 'spatial models', a tradition of geometric formalization applied originally to electoral competition but which is of more general relevance, increasingly providing alternative formalizations of important political concepts, including such game-theoretic concepts as 'conflict of interest' (see also Farquharson, 1969).

The articles assembled under the heading 'power and policy success' together are formally complementary to game theory. They variously formalize some of the dynamics of decision-making and in particular formally demonstrate an evolutionary tendancy for decision-making bodies to approach the small-N coalition situation (Ch. III).

The articles on the 'vote/seat relation' attempt to formally specify a number of concepts in political science relating to electoral systems, thus complementing social choice theory. One of the articles is formally functional (Theil, 1969) and I have relied heavily on the others in the construction of the preliminary functional model (Ch. V).

V. CONCLUSION

A. Introduction

The purpose of the thesis has been to contribute toward the development of a formal theory of the political system. I have reviewed the current state of functional theory of politics. In this chapter I shall first summarize my assessment of the current state of functional theory of politics. Then I shall outline the direction which I think further work should take. I see this as consisting of three phases: (1) the partitioning of the general domain of politics into special domains, each defined by one essential variable; (2) construction of special functional theories, each explaining the special domain as a functional subsystem, with management of its essential variable (toward some extremum) as the theoretical principle; (3) the construction of a general functional theory of political action, with management of the relations between the essential variables as the theoretical principle.

In this chapter, I shall first summarize the results of my review of the literature. I have defined a formal method of theory construction (formal functionalism) and surveyed some relevant formal literature in political science for functional formulations or for material which may be amenable to a formal functional treatment. I shall then discuss the possibility of the construction of a formal functional theory in light of the assessment in the

following order: (1) the nature of *functional theory*, (2) the key idea of an *essential variable*, (3) an example of a special theory, and (4) the ultimate end of a *political action theory*.

Although such a theory has not been fully accomplished in the political science literature, I shall set out a preliminary special theory, in which are defined one essential variable and the functional subsystem variables which act to maintain it at an extremum. I shall first formulate *power*, which seems to be most clearly addressed in the literature, as an essential variable, then outline a set of variables extracted from the literature which act to maintain the variable at an extremum, and note the condition of functional stability which results.

I shall then outline a 'conceptual sketch' of other possible essential variables; that is, variables which could be candidates for similar treatment and which taken together could ultimately result in a general theory of political action. Finally I shall briefly discuss the possible future general theory of political action.

B. Political Science As Functional Theory

I shall here summarize the results of my review of the current state of functional theory of politics.

In **chapter** 1, I argued that the *'functional tradition'* in sociology and political science was inadequate with respect to the development of a formal functional theory.

While addressing key concepts, traditional functionalism retreats from a rigorous logical treatment and instead engages in a metaphorical elaboration of the concepts. It was also noted, however, that one can distinguish certain formal distinctions inhering in the vast verbal metaphors of the functional tradition. These are the partition of concepts into what can be interpreted as essential variables (whether characterized as goals, purposes, or 'functional prerequistes' such as leadership) answering to the primary concerns to be explained in their work, a set of variables which act to maintain some state of the important concept, and a number of other other variables which act as bounds to the system so maintained.

Turning to more highly formalized literature in political science, in chapter 2 I examined the *theory of social choice*. Social choice theory formalizes a clearly political situation, namely the aggregation of choices or preferences to some social state, both the aggregation process and the result being subject to certain formal conditions.[87] Furthermore the aggregation process is formally functional in nature; it is a maximization problem, indeed a mathematical functional. In addition, the formally embodied conditions can be seen as institutionalized values, variously interpretable as other 'essential' aspects of politics or as boundary conditions to social decisions.

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[87] I argue that the variables 'power' and 'policy' summarize what gets aggregated in social choice theory.

Social choice theory does not, however, offer a complete formal theory of politics. I discussed violations of the 'value' conditions in real political systems, the inadequacy of certain 'rational' assumptions in the light of a wider context of political action and actor types, the failure to address 'game' features of political action (see Ch. III), the failure to formally embody the necessity for stability of decisions, and the assumptions of legitimacy and enforcement (see below) of social choices.

Even where inadequate, though, the social choice model of political action offers formally stated departures from which to develop a more complete model.

The literature of *game theory* (Ch. III) even more clearly formalizes features which are at the heart of or are the 'essence' of politics. Game theorists have explored a wide variety of political situations: the many distinct game types allow formalization of such features as cooperation, information conditions, number of actors and so on. In particular, those aspects of 'strategic interaction' ignored by social choice theorists are formalized in game theory. Also, game theory is formally functional in nature, and although static in formulation seems amenable to dynamic elaboration (see Axelrod, 1970; Schofield, 1978).

I argued for the particular importance of the N-person game as a formal framework for the analysis of politics. Much work has been done in this 'coalition theory' particularly with political parties considered as political

actors: these actors typically form coalitions. The formal search is for 'equilibrium' coalitions, a patently functional notion. Indeed, in coalition theory one has virtually a formalized essential variable in the construct 'mimimum winning coalition'. An 'MWC' exhibits the formal characteristics of an essential variable, and there is even empirical support for its maintenance (see Ch. IV). The construct 'MWC' is the end result of a trend in the search for stable coalition towards the aggregation of both of what I have called 'power' and 'policy' (see Ch. III). Game theory probably offers the best formal framework for a functional political theory.

In **chapter 4**, I considered a variety of mathematical approaches which have been used in addressing problems of political/electoral systems. These offer alternative formalizations of important issues in politics.

The '*spatial models*' offer a geometric formalization of the 'policies' of both political parties and individuals. The political action involved is typically the competition between parties to maximize their support in some electorate. This formalization has been applied to a number of political issues, for example the relation between the number of parties and the types of policies. Most important perhaps, a spatial model of 'conflict of interest' (based on the idea of a party ideological spectrum) has been used (Axelrod, 1970) to test a variety of coalition theories (see Ch. III).

In the next section of chapter 4, I examined various formalizations of the problem of '*power and policy success*' in committees and legislative bodies. I argued that these formalizations complement the work in game theory; in particular they collectively provide evidence for the special relevance of the coalition situation. That is, the articles give independent formal support for the importance of the 'minimum winning coalition': I believe that this literature clearly and mathematically demonstrates that one should, on formal grounds, expect a transition to the coalition situation and that this constitutes 'explanatory' support for the existence of 'blocs', factions, and political parties in developed political systems. At the same time the results add a dynamic element, by showing how this essential variable evolves in political activities.

The articles on the '*vote/seat relation*' offer alternative formulations of the aggregation of individual choices into a social decision; they are therefore complementary to the work on social choice theory. Starting from important political issues, they attempt to formalize their essential elements, and have arrived at mathematical explanations of 'proportional' versus 'plurality' electoral systems, the stability of two-party systems and the occasional rise of third parties. Theil (1969) has set out a general formalization of all possible such aggregation systems which would appear to have functional elements.

In summary, the studies reviewed have provided a number of elements suitable for the development of a formal functional theory of politics. These elements are both formal and conceptual. For, besides providing 'bits of structure' or formal constraints which can be built into the formal functional system, these studies are an attempt to extract and formalize essential concepts of politics. The work here is for the most part conceptually interpreted; one has in fact some ready 'laws' in the literature. More than this, the two major bodies of literature include definitions of political systems, however inadequate they may be as total conceptions of politics.

The need is to turn this formal and conceptual material to the uses of functionalism.

C. Formal Functionalism

I shall now disuss, in the light of my assessment, the possibility of the construction of a formal functional theory.

Formal functional theory would explain the operation of political systems to the extent to which it could identify (a) extremum values of essential variables, (b) functional subsystems determining the values of the essential variables, and (c) the boundary conditions under which the functional subsystems maintain the essential variables at the extremum values.

Essential Variables

The first task in the construction of a functional theory is to identify particularly salient features of politics and to enquire as to how these might be formulated as essential variables; that is, theoretical (formal and conceptual) constructs with specified extrema (Jung, 1973a).

From the literature one can extract the importance of certain key aspects of politics. These are clearly recognized as such in the literature and in political life alike. They partake of or form the *essence* of political action. It is recognized that without their maintenance one would not really have politics. Such key aspects of politics would be, for example, conflict, cooperation, decision-making, regulation governing conflict, the legitimacy of policy-forming procedures, and the need to transmit decision into action. These key aspects of politics are the source of 'essential variables'.

Special Theories of Action

In addition to the essential variable 'power', one can extract from the literature other key or defining variables of politics; these variables are either a direct focus of interest in much of the literature (for example, the concern with preference, best choices, and 'policy success') or are assumptions within which the work operates (for example legitimacy of the rules of operation is assumed in all the work here discussed, and is directly addressed in social

choice theory).

The future aim should be the creation of 'special' functional theories for each of the essential variables. The special theories would consist of the essential variable formally constrained within a critical interval and the subsystem variables which act to maintain the constraint. In this each special theory would explain an important part of the total realm of political action.

General Theory of Political Action

A further stage in a functional approach to politics would be the construction of a functional model for not one but for all the essential variables of politics. Such a theory would be a 'general' theory or model of political action. Within this general model would be the 'special' theories for the management of each particular essential variable of politics. This political action model would show how the political system *manages the relations between the essential variables*. The essential variables would be related to each other, perhaps acting as 'conditions' for each other. It is these relations between essential variables that a general theory of political action would explain.

Such a general model has not been accomplished in the formal political science literature; such attempts as have been made are all in the 'metaphorical' tradition. However, there is in the literature sufficient material for the

possible construction of a 'special' theory, concerned with one essential variable. Below, I shall discuss a 'special' theory for the variable which I have called 'power'. I shall define the variable, outline the subsystem variables and their relations, and show how these act to maintain the essential variable at a critical interval.

Such could in principle also be done for the other variables, but I have not been able to formulate such special theories from the literature I have surveyed. However, I have set out a 'conceptual sketch' of variables which seem to embody basic or essential characteristics of politics. These would be 'candidates' for treatment as essential variables in any further construction of a general theory of political action.

D. A Preliminary Functional Model

Introduction

In this section I will present an early and unrevised model,[88] consisting of some variables in the political science literature ordered in the light of a functional mode. This will include an essential variable, relations involving functional subsystem variables, and an attempt to show that these relations operate to maintain the essential

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[88] The model is the result of first attempts to develop a functional theory of politics, based on a search of political science literature and the variables discussed there. This survey is meant to begin the development of a model which remedies its formal, conceptual and empirical inadequacies.

variable at an extremum value.

The relations to be discussed are both formal and conceptual; that is, they are at the level of theory. However, the formalization is not sufficient to allow the specification of propositions to be empirically tested. The relations are more in the nature of 'explanatory principles' (Jung, 1974) than laws, ruling out some conceivable relations while selecting families of propositions.

The empirical adequacy of the relations may be questioned. Since this is a theoretical discussion, it is of less concern than otherwise that the relations be empirically or inductively adequate. At this point I am satisfied that the relations have some currency in the literature.[89]

The Essential Variable: Power

There is clearly much anecdotal support for the view that *power* is an essential aspect of the political system.

That this variable can be used to handle salient aspects of politics can be seen in its assumed importance to a variety of students, in its adequacy with respect to definitions of politics, in actions taken when the value is not maintained and in the stability political sytems show in maintaining the critical value and in returning to it from

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[89] For example, the fact that 'Duverger's Law' has exceptions will not deny validity to the analysis; Rae's (1967) application of the criterion of disconfirmation to this law is inappropriate at this level. Furthermore, the theoretical question would be whether the exceptions could themselves be formalized.

deviations (Pinard, 1971; Spafford, 1972). That is, the variable has both conceptual and empirical support. But the relevance of power in the above sense can also be seen in the possibility of summarizing significant aspects of the formal work previously discussed; that is, significant aspects of social decision and game theoretic work can be interpreted as relating to the maintenance of this essential variable.

The political system must both accomodate and reduce complexity with respect to action on political issues (Luhmann, 1974). It accomodates complexity by structurally guaranteeing uncertainty: through elections, party alternatives, coalition possibilities and other institutional flexibilities. It reduces complexity by ensuring that issues can be decided. A key feature is that the system ensures decisions not for single issues, but for a range of issues, which issues may not be known in advance (Luce & Raiffa, 1957).

Definition

Power then is an essential variable; power in the sense of having sufficient support to pass issues, a relatively safe majority or, in game theoretic terms, a 'winning coalition' under some decision rule.[90]

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[90] The decision rules embody values with relevance to the particular institutional setting of the decisions being made. For example, the 'rule of unanimity' in juries formally embodies the legal principle that 'It is better a hundred guilty men go free than that one innocent man be convicted'. So the struggle for power takes place in the context of institutionally embodied values, as decision

[90] In these terms the political system is a mechanism for aggregating sufficient voting power into some power-bloc in a decision-making body to allow the relatively certain passage of decisions it proposes.

Extremum values

Formally, there is an upper as well as a lower bound to the critical value of the variable. That is; the lower bound being approximately the power necessary to just pass laws, the upper bound is *less* than all the voting power in the decision-making body. In game theoretic terms it is a *mimimum* winning coalition.[91] The variable *power* thus exhibits the formal properties of an essential variable.

To summarize, the political system operates to maintain the variable *power* at a critical value with a lower bound of just sufficient votes to carry issues and with a less clear-cut upper bound significantly less than the total voting strength of the assembly. This value of the variable is maintained through the relations of variables in the functional subsystem (described below) and within bounds of variables acting as boundary conditions.

Functional Subsystem Variables

Proportional/Non-proportional Systems

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[90](cont'd)rules. These might be conceived as Vbc's to the struggle for power.

[91] The presence of this upper bound is implied in the work on coalition theory (Ch. III) and, in the pluralistic context, in the phenomenon of the 'rise of third parties'(Ch. IV).

One can discern a rough configuration of the values of a number of politically relevant variables into two broad types: proportional and non-proportional (or plurality) systems. The variables are such properties in the literature as vote/seat relation, voter turnout, constituency size, bases of party appeal and others (see Ch. IV, the 'cube law'). The distinction between proportional and non-proportional is both a type in its subsumption of such variables and a variable itself with these two values. Assuming this typology makes the following discussion of relations simpler and excuses to some extent the failure to consider many variables.

The Number Of Parties

The values of this variable are the number of parties receiving significant electoral support, with deliberate vagueness as to levels necessary for significance.

Coalition Formation

Some political systems regularly have governments formed from two or more parties; others seldom have. The values are the number of parties forming the government, 'one' being a non-coalition government.

Party Discipline

This variables has higher or lower values primarily according to the extent members vote together on issues in the decision-making body.

Functional Subsystem Relations & the Essential Variable

P/Non-P Systems & the Number of Parties

According to 'Duverger's Law' (1951/1959) systems with non-proportional voting tend to become two-party systems while proportional systems tend to several parties. One explanation for this, based on individual voter assumptions and on structural features (e.g. the seat/vote relation) is characterized as the 'Wasted Vote Thesis'.[92]

This relation has considerable inductive support (Rae, 1967) and there even exists in the operation of the West German political system quasi-experimental support (Fisher, 1973).

There are some structural conditions dealing with exceptions to the law, in particular that strong party competition is a necessary condition for two parties (Spafford, 1972) or that one party dominance is a sufficient condition for the rise of a third party, thus negating the law (Pinard, 1971). These exceptions can be interpreted in terms of the essential variable. One party dominance is equivalent to a value on the essential variable *above* the critical value; the rise of a third party brings the value

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[92] The wasted vote thesis posits that, in pluralist systems, where only the first party in a constituency gets a member elected and, as a consequence, small parties are much underrepresented, voters will not vote for their preferred party if it is small because the vote doesn't really 'count'. Hence small parties do even worse.

A 'boundary condition' to this is the fact that the 'cube law' doesn't operate when there is strong geographic clustering in voting support. In this case, small 'regional' parties can be greatly *over*represented.

back within the critical value.

P/non-P Systems & Coalition Formation

Non-proportional political systems commonly have no coalition formation among parties, one party forming the government. Proportional systems regularly have two or more parliamentary parties in the government.

One explanation for this feature is the 'size principle' (Riker, 1962), which can be interpreted as approximately equivalent to the maintenance of the essential variable within the critical value. The size principle can be seen in operation in proportional systems directly, in coalition formation. In non-proportional systems, the variable is maintained through reduction in the number of parties (increasing the likelihood of majorities for one party) and also by the operation, in such systems, of the 'Cube Law' (Kendall & Stuart, 1950) which produces 'artificial majorities' in absence of majority support.

Empirically, exceptions to this relation, while not rare, seem relatively unstable and tend to evolve to a situation in which the essential variable is within the critical interval - neither overwhelming majorities (grand coalitions) nor minority governments last.

P/Non-P Systems & Party Discipline

As the number of parties increases, there is a rough increase in the level of party discipline. An intuitive explanation for this is the idea that parties divide some range or spectrum of conflict of interest (Axelrod, 1970):

party formation establishes (and exaggerates) boundaries within the range. Many parties is equivalent to 'streaming' in contrast with the 'comprehensive' nature of a few parties. Increasing the number of parties exaggerates external boundaries and suppresses internal divisiveness; decreasing the number of parties reduces the number of external boundaries and divisions are increased internally.

Some support for this interpretation is given by Lipset (1960) in his discussion of the narrowly based and intense appeals during campaigns among multi-party systems.

Relating this to the P/Non-P variable, it is notable that proportional systems require high levels of party discipline; party lists exert strong control over individual members, and multi-member constituencies diminish the direct influence of voter opinion on members.

Explained in terms of the essential variable, proportional systems require parties to 'act as one' during the necessary coalition negotiations and decision-making; high discipline is essential. One would expect high discipline in non-proportional systems as well, when party strength is close to the lower bound of the critical value (i.e. a bare majority) and relatively low discipline when there are large majorities. There is some support for this (Riker, 1962).

Functional Stability

One can envisage the political system to consist of the set of variables discussed, which variables together act to maintain a 'working majority' (winning coalition) in a legislative assembly. Proportional systems with many parties have a process of coalition formation in order to reach this working majority; plurality (non-P) systems with few parties have no such process, for the plurality system produces 'artificial' majorities for the party with the largest vote. Under the 'perturbations' of expressions of varied opinions at elections, each channels the opinion differently to the *same* end.

Both systems are democratic - but *where* the democracy occurs differs. In one case opinion is accurately represented at the voting level, in parties and seats, but the coalition process takes place after the voting stops. In the other case, votes are seriously distorted but the plurality party achieves governance with rapidity and certainty on election night. In any case both systems produce a minimum winning coalition.

One can also discern that this majority is an evolutionary end: beginning with a 'random' and varied collection of individual preferences the result is the formation of blocs (including parties) until the majority is reached (see Ch. IV).

There is further evidence of action in political systems to *restore* minimum winning coalitions when the

'critical interval' is violated.

And there are boundary conditions, such as the geographic dispersal of votes, to the operation of the system.

E. Essential Variables of Politics

I will in this section attempt a preliminary 'conceptual sketch' of several essential variables of politics. The conceptual sketch will make use of the previously discussed work. Together the essential variables would 'partition' politics into its key areas, each essential variable explaining the operation of a further subsystem of variables which act to maintain it at a critical interval.

The above comments are, of course, preliminary. At least the following problems will need further consideration.

Formal

The formal structure of the variables will need to be specified. In particular this relates to the level of formalization and to the formal extremum properties. The utility of conceiving the variables as probabilistic should be considered.

Empirical

The extent to which the variables are really positive features of political systems must be considered, particularly the pattern of any deviations from the critical

interval.

Conceptual

The choice of conceptual variables inevitably has arbitrary aspects. The posture taken here is that important aspects of political systems could be handled through the essential variables selected. The formal and empirical importance of the variables will remove some of this arbitrariness; the further possibility of independent conceptual criteria of importance should be considered. Such criteria might derive from consideration of properties especially 'meaningful' to political actors, or from the relation of such concepts to phenomenologically derived concepts at the individual level, as well as the formal and conceptual literatures.[93]

In this 'conceptual sketch' I shall discuss as candidates for essential variables of politics *policy*, *legitimacy*, *enforcement* and *sovereignty*. I have already considered the variable *power* (see above, part D).

Policy

The output of any political system will be decisions with respect to issues; that is, the political system will be structured to handle policy.

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[93] For example, the power and policy essential variables might be conceived as the aggregate equivalent of probability and desirability, the variables which define the individual decision space. One variable would be the probability of power, the other would be aggregate evaluative structure.

Some policy variable or variables is intuitively necessary to catch essential features of political systems. And in the previous formal work policy considerations are major elements. It is clear that individual preferences are both power (votes) and policy (orderings). The basis for social decisions is the 'issue universe' or 'policy space'. And in coalition theory even the most power oriented students of politics are forced to go beyond the size principle and grant policy a role in in determining stable coalitions; instability of the power variable about its critical value can only be handled through policy considerations (Riker, 1962).[94]

Definition

I am at this point less clear about the formal features of the policy variable; for individuals one might expect some maximization of policy correspondence to individual policy spaces (Rae, 1969). For political systems, which consist of individuals and parties with contrary policies this is unlikely. Certainly the policy variable will interact with the power variable; it may indeed be better to conceive the essential variable as a combination of the two; as some relational variable, the structure being the relation between power and policy.

Legitimacy

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[94] Indeed, much of the work reviewed, especially the voting theory, can be interpreted as a policy or opinion aggregating system, as well as a power aggregating system.

The political system normally operates within a framework of legitimacy. Issues or policy over which conflict occurs do not include the basic values of political rule, and only rarely the 'ground rules' under which the decisions are made. In a phrase the political system operates in a 'legal order'.

That legitimacy of the system might well be regarded as an essential variable of politics receives support both from the fact that this variable is maintained in the normal course of events and from the existence of efforts to regain legitimacy when it is lost. Indeed, much of political life is involved with the maintenance of this legitimacy; conflict over issues often also includes affirmation or denial of legitimacy. A party or group of parties which does not grant legitimacy to the system under which it operates makes the maintenance of the political structure very precarious.

Definition

While this is often assumed in the literature, I would treat legitimacy explicitly, regarding it as either another essential variable of the political system or as a boundary condition to the operation of a power/policy aggregation system.[95]

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[95] Power and legitimacy are conceptually separate although they may interact. One can have power without legitimacy when, for example, a military or civil service organization accomplishes its ends outside of legitimate channels. In such a case the 'proper' channels would exemplify the converse - legitimacy without power. Indeed, as with power and policy, one might consider the electoral system to be a

[95]

I have yet to determine the formal characteristics of this variable; it may be that the required extremum would take the form of some minimum set of procedures in making decisions. That is, output decisions are rendered legitimate through their being produced in a structure which gives present or future decisions their legitimacy or legality (Luhmann, 1974). It is the structure which is legitimate (Jung, 1981). Perhaps one can view this formally as certain minimum conditions which are necessary to make an decision or action legitimate: one must accomplish the policy a certain way.

And some noted features of politics can be accounted for by invoking legitimacy; for example, the lack of formal coalitions in non-proportional systems even when there is a minority government; that is, when considerations of power alone dictate coalition, can probably be explained in terms of the lack of legitimacy of such 'blatant' collusion among groups ('her majesty's loyal opposition') whose *duty* is to oppose.

Enforcement

The production of decisions about policy in a political system obviously occurs with a view to achieving intended effects. Yet one must distinguish a decision with respect

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[95](cont'd)'legitimation' machine.

Legitimacy with power would be an 'authority' system. The examples in the literature are all of legitimate power; however the two concepts should be clearly distinguished.

to some policy from the action of carrying out the policy. In the literature it is occasionally assumed that, in the generalized leadership the political system provides (Lipset, 1960), accomplishing the decision accomplishes the fact. This may not be. The political system interacts with other systems; there will be a varying relation between decision and the actual enforcement of policy. A decision may be made but not translated into action; conversely, actions occur in the absence of decision, or in the face of contrary decisions.[96] A 'successful' political system must be able to decide policy, and be able to enforce it within its sphere of sovereignty.

Definition

Given this distinction it is at least clear that some concept of enforcement or regulation can be plausibly considered essential to political systems. Formally, I have yet to determine the structure of the variable. It could perhaps take the form of some minimum probability of translating decision into action.

Sovereignty

Other systems in a society exhibit some or all of the properties so far discussed. For example, the judicial

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[96] Consciousness of the first possibility; that is, an expected low probability of translating decisions into action, may indeed place prior restrictions on policy: whole regions of the 'policy space' for decision may be ruled out (Jung, 1981). A nation's international position is often of this character. In the second case, when interacting systems accomplish actions which are in the sphere of legitimate decision, but without such decision, the political system may simply function to legitimize the act.

system possesses power and legitimacy.[97] Sovereignty, however, is unique to political systems (Crick, 1962).

I am uncertain as to both the conceptual nature and the formal properties of this variable. The effect seems to be to generalize the action of the other variables; sovereignty gives the political system absolute domain, in a sense, over other systems. Looked at from outside, sovereignty establishes the scope of political action.

Along with legitimacy, sovereignty is ignored (or assumed) in the formal political theory discussed.

One could 'carve up' the domain of politics in other ways, as has been noted. For example other variables might be considered essential.[98] Such consideration would obtain in the further conceptual development of the theory.

F. General Theory of Politics

The ultimate end of further work would be the successful construction of a theory of political action. Upon the prior partition of the domain of politics into essential variables, each formally and conceptually encompassing some essential aspect, and the construction of

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[97] One can view institutions in society in terms of combinations of these essential variables. Other systems in society, for example, the economic system, have power: the political system has power *with* legitimacy. The judicial system, in turn, has both power and legitimacy, but, unlike the judiciary, the political system can make policy.
[98] A variable 'territoriality' which would organize the varied overlapping jurisdictions of municipal, 'state' and federal systems, might justifiably be said to conceptualize an essential aspect of politics.

special functional theories, in which a subsystem of political variables are shown to vary so as to maintain each essential variable at an extremum, such a theory of political action could be constructed.

It would consist of an overarching or general theory, within which the special theories would be accommodated. The general theory would formalize the entire domain of political action by explaining the relations between the essential variables of politics.

No such theory has been developed. The conceptual sketch and 'special' theory mark only a beginning, and the survey upon which it is based should be broadened. The work reviewed offers formal definitions of politics in which the concepts power, policy and legitimacy are entwined; in particular the view of power is restricted in the social choice and voting theory.[99] Further development would require a systematic conceptual analysis of politics, the aim of which would be clearly (conceptually, formally, empirically) delineated variables, allowing the explanation of subsystem variables and the formalization of explicit relations between the variables in a general theory of politics.

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[99] Coalition theory does, however, offer the possibility of formally extending the concept of power beyond the legitimate electoral situation.

G. Summary and Conclusion

The purpose of the thesis has been to contribute toward the construction of a formal theory of the political system. In chapter 1, I outlined a method for developing a 'formal functional' theory of politics, the first task of which would be the identification of important concepts or properties of politics and their formulation as essential variables.

This survey (Chs. II to IV) is preliminary to the construction of such a theory. Its purpose has been to canvass the literature for formal structures (and associated concepts) which offer the possibility of functional formulation. The literatures of social choice and game theory have been shown to be rich in mathematical structures, some of which seem functional in nature, and there have been mathematically embodied concepts, in particular the construct 'minimum winning coalition', which seem to catch essential aspects of political action. In this chapter (V), I have assessed the current state of functional theory of politics, and constructed a preliminary functional model and 'conceptual sketch' of the political system from the reviewed literature. I have also outlined the possible direction of future work including the construction of a general theory of political action.

BIBLIOGRAPHY

Ackoff, R. L., & Emery, F. E. *On purposeful systems.* New York: Aldine-Atherton, 1972.

Alker, H. R., Jr., Deutsch, K. & Stoetzel, A. H. *Mathematical approaches to politics.* San Fransisco: Jossey-Bass, 1973.

Allen, R. *Mathematical analysis for economists.* London: Macmillan, 1938.

Almond, G. A. Introduction: a functional approach to comparative politics. In G. A. Almond & J. S. Coleman (Eds.), *The politics of developing areas.* Princeton, N. J.: Princeton University Press, 1960.

Aranson, P. H., & Ordeshook, P. C. Spatial strategies for sequential elections. In Niemi, R. H., & Weisberg, H. F. (Eds.), *Probability models of collective decision-making.* Columbus, Ohio: Charles E. Merrill, 1972.

Aristotle. *The politics of Aristotle.* (T. A. Sinclair, trans.) Markham, Ontario: Penguin Canada.

Arrow, K. J. *Social choice and individual values.* New York: John Wiley & Sons, 1963.

Ashby W. R. *Design for a brain.* London: Chapman & Hall, 1952.

Ashby, W. R. *An introduction to cybernetics.* London: Chapman & Hall, 1956.

Axelrod, R. *Conflict of interest.* Chicago: Markham, 1970.

Bacharach, M. *Economics and the theory of games.* London: Macmillan, 1976.

Baker, M. *Condorcet: From natural philosophy to social mathematics.* Chicago: University of Chicago Press, 1975.

Bellman, R. *Mathematical theory of control processes.* New York: Academic Press, 1967.

Berger, J., Cohen, B., Snell, J., & Zelditch, M. *Types of formalization in small group research.* Boston: Houghton-Mifflin, 1962.

Berlinski, D. *On systems analysis: an essay concerning the limitations of some mathematical methods in the social, political and biological sciences.* Cambridge: MIT Press, 1976.

Bernoulli, J. On the brachistochrone problem (L. LaPaz, trans.). In Smith, D. E. (Ed.), *Sourcebook in mathematics.* New York: McGraw-Hill, 1929.

Black, D. *The theory of committees and elections.* Cambridge: Cambridge University Press, 1958.

Black, M. *The social theories of Talcott Parsons.* Englewood Cliffs: Prentice, 1961.

Black, M. *Models and metaphors.* Ithaca, N. Y.: Cornell U. P., 1962.

Borel, E. The theory of play and integral equations with skew symmetric kernels (L. Savage, trans.). *Econometrica,* 1953, *21,* 97-100 (Originally published, 1921).

Braithwaite, R. *Theory of games as a tool for the moral philosopher.* Cambridge: Cambridge U.P., 1955.

Brams, S. J. *Game theory and politics.* New York: Free Press, 1975.

Buckley, W. *Sociology and modern systems theory.* London: Prentice-hall, 1967.

Cannon, W. B. *The wisdom of the body.* New York: W. W. Norton, 1932.

Cartwright D., & Harary, F. Structural balance: a generalization of Heider's theory. *Psychological Review*, 1956, *63*, 277-293.

Chaing, A. *Fundamental methods of mathematical economics.* Toronto: McGraw-Hill, 1974.

Coleman, J. Foundations for a theory of collective decisions. *American Journal of Sociology*, 1966, *71*(May), 615-27.

Courant, R. Mathematics in the modern world. *Scientific American*, 1964, *211*, 41-49.

Crick, B. *In defense of politics.* Harmondsworth, England: Penguin Press, 1962.

D'Abro, A. *The rise of the new physics.* New York: Dover, 1951.

Dahl, R. A. *A preface to democratic theory.* Chicago: University of Chicago Press, 1956.

Davis, O. A., & Hinich, M. J. Some results related to a mathematical model of policy formation in a democratic society. In J. L. Bernd (Ed.), *Mathematical*

applications in political science (vol. 3). Charlottesville: University Press of Virginia, 1967.

Davis, O. A., Hinich, M. J., & Ordeshook, P. C. An expository development of a mathematical model of the electoral process. *American Political Science Review*, 1970, *64*, 426-428.

Davis, O. H., & Hinich, M. J. A mathematical model of policy formation in a democratic society. In J. L. Bernd (Ed.), *Mathematical applications in political science* (Vol. 2). Dallas: Arnold Foundation, 1966.

DeSwann, A. An empirical model of cabinet formation as an n-person game of policy distance minimization. In S. Groennings, E. W. Kelly & M. A. Leiserson (Eds.), *The study of coalition behavior*. New York: Holt, Rinehart & Winston, 1970.

DeSwann, A. *Coalition theories and cabinet formations*. London: Elsevier, 1973.

Diesing, P. *Patterns of discovery in the social sciences*. New York: Aldine-Atherton, 1971.

DiStefano, J.; Stubberud, A. R.; & Williams, I. J. *Feedback and control systems*. Toronto: McGraw-Hill, 1967.

Dodd, L. C. Party coalition in multi-party parliaments: a game theoretic analysis. *American Political Science Review*, 1974, *70*, 1093-1117.

Downs, A. *An economic theory of democracy*. New York: Harper, 1957.

Duverger, M. *Political parties* (rev. English ed., B. & R. North, trans.). London: Methuen, 1959. (Originally published, 1951)

Easton, D. *A framework for political analysis.* Englewood Cliffs, N. J.: Prentice-Hall, 1965. (a)

Easton, D. *A systems analysis of political life.* New York: John Wiley & Sons, 1965. (b)

Emery, F. E. *Systems thinking.* Harmondsworth, England: Penguin, 1969.

Emmet. D. *Function, purpose and powers.* London: the Macmillan press, 1958.

Engelmann, F. C., & Schwartz, M. A. *Political parties and the Canadian social structure.* Scarborough, Ontario: Prentice Hall, 1967.

Farquharson, R. *Theory of voting.* New Haven: Yale University Press, 1969.

Fishburn, P. C. *The theory of social choice.* Princeton: Princeton University Press, 1973.

Fisher, S. The wasted vote thesis. *Comparative Politics,* 1973, *5,* 293-299.

Formby, J. *An introduction to the mathematical formulation of self-organizing systems.* London: van Nostrand, 1965.

Forsyth, A. R. *Calculus of variations.* New York: Dover, 1960.

Frank, L. K. Forward. In *Teleological mechanisms* (Annals of the New York academy of sciences). 1948, *50* 187-195.

Gaines, B. Foundations of fuzzy thinking. *International*

journal of man-machine studies, 1976, *8*,, 623-668.

Gale, G. The anthropic principle. *Scientific American*, 1981, *245* (Dec.), 154-165.

Groennings, S. Notes toward theories of coalition behavior in multi-party systems: formation and maintenance. In S. Groennings, E. W. Kelly & M. A. Leiserson (Eds.), *The study of coalition behavior*. New York: Holt, Rinehart & Winston, 1970.

Halevy, E. *The growth of philosophic radicalism*, (M. Morris, trans.). London: Faber & Faber, 1972.

Harris, R. J. A geometric classification system for 2X2 interval-symmetric games. *Behavioral Science*. 1969, *14*, 138-146.

Hays, W. H. *Statistics for the social sciences*. Toronto: Holt, Rinehart & Winston, 1973.

Henderson, L. J. *Pareto's general sociology: a physiologist's interpretation*. New York: Russell & Russell, 1935.

Henderson, L. J. *On the social system* (B. Barber. ed. & intro.). Chicago: University of Chicago Press, 1970 (Originally published, 1927-1942).

Hermens, F. A. *Democracy or anarchy: a study of proportional representation*. South Bend: University of Notre Dame Press, 1941. Russell & Russell, 1935.

Hotelling, N. Stability in competition. *Economic Journal*, 1929, *39*, 41-57.

Howard, N. *Paradoxes of rationality: theory of metagames*

and political behavior. Cambridge: the MIT Press, 1971.

Howard, N. General metagames: an extension of the metagame concept. In A. Rapoport (ed.), *Game theory as a theory of conflict resolution.* Dordrecht, Holland: D. Reidel Pub. Co., 1974.

Inada, K. *On the economic welfare function* (Technical report No. 97). For the Office of Naval Research, Institute for Mathematical Studies in the Social Sciences, Stanford University, Stanford California, 13 July, 1961. (Contract Nonr-225(50))

Isard, W. *General theory: Social, political, economic, and regional, with particular reference to decision-making analysis.* Cambridge, Massachusetts: The MIT Press, 1969.

Jung, R. Self-control in a sociological perspective. In S. Z. Klausner (Ed.), *The quest for self-control: Classical philosophies and scientific research.* New York: The Free Press, 1965. (a)

Jung, R. Systems of orientation. In M. Kochen (Ed.), *Some problems in information science.* New York & London: The Free Press, 1965. (b)

Jung, R. *Types of decision and intervention.* Paper presented at the Airlie House Conference on Social Conflict, Warrington, Va., June, 1965. (c)

Jung, R. *Cybernetic phenomenology.* Paper presented at the meeting of the American Sociological Association, Denver, August, 1971.

Jung, R. *Modes of explanation.* Lectures presented at the University of Alberta, Department of Sociology, Fall, 1973. (a)

Jung, R. *Decision theory.* Lectures presented at the University of Alberta, Department of Sociology, Fall, 1973. (b)

Jung, R. *The basic language of science.* Lectures presented at the University of Alberta, Department of sociology, 1974.

Jung, R. Lectures presented at the Center for Advanced Study in Theoretical Psychology, University of Alberta, Fall, 1976.

Jung, R. Personal communication, 1981.

Kemeny, J. G., Snell, J. L., & Thompson, G. L. *Introduction to finite mathematics.* Englewood Cliffs, N. J.: Prentice-Hall, 1957.

Kemeny, J. G. Mathematics without numbers. *Daedalus*, 1959, *88*, 577-591.

Kendall, M. G., & Stuart, A. The law of the cubic proportion in election results. *British Journal of Sociology*, 1950, *1* (3), 183-196.

Koestler, A. *The act of creation.* New York: Macmillan, 1964.

Krupp, S. R., Equilibrium theory in economics and functional analysis as types of explanation. In D. Martindale (Ed.), *Functionalism in the social sciences* (Monograph 5). Philadelphia: The American Academy of Science,

1965.

Langer, S. K. *An introduction to symbolic logic* (rev. 3rd ed.). New York: Dover, 1967.

Lee, D. Analogy in scientific theory construction. In *Southern Journal of Philosophy*. 1969, Summer, 107-115.

Leiserson, M. A. Factions and coalitions in one-party Japan: An interpretation based on the theory of games. *American Political Science Review*, 1968, *62*, 770-787.

Leiserson, M. A. Coalition government in Japan. In S. Groennings, E. W. Kelly & M. Leiserson (Eds.), *The study of coalition behavior*. Toronto: Holt, Rinehart & Winston, 1970. (a)

Leiserson, M. A. Game theory and the study of coalition behavior. In S. Groennings, E. W. Kelly, & M. Leiserson (Eds.), *The study of coalition behavior*. Toronto: Holt, Rinehart & Winston, 1970. (b)

Leiserson, M. J. Power and ideology in coalition behavior: An experimental study. In S. Groennings, E. W. Kelly, & M. Leiserson (Eds.), *The study of coalition behavior*. Toronto: Holt, Rinehart & Winston, 1970. (c)

Lipset, S. M. Party systems and the representation of social groups. *European Journal of sociology*, 1960, *1* (1), 50-85.

Lipson, L. *The great issues of politics* (3rd ed.). Englewood Cliffs, New Jersey: Prentice-Hall, 1965.

Lopreato, K. A functional reappraisal of Pareto's sociology. *American Journal of Sociology*, 1964.

Luce, R. D., & Raiffa, H. *Games and decisions.* New York: Wiley, 1957.

Luhmann, N. Sociology of political systems. In K. von Beyne (Ed.), *German political studies.* New York: Sage Publications, 1974.

March, J. G. Party legislative representation as a function of election results. *Public Opinion Quarterly*, 1957, *21*, 521-542.

May, K. O. A set of independent necessary and sufficient conditions for simple majority decisions. *Econometrica*, 1952, *20*, 680-684.

Malinowski, B. Anthropology. *Encyclopedia Britannica* (Suppl, Vol. 1). New York: 1936.

Martindale, D. *The nature and types of sociological theory.* Boston: Houghton Mifflin, 1960.

McGinnis, R. *Mathematical foundations for social analysis.* Indianapolis: Bobbs-Merrill, 1965.

McKinsey, J. *Introduction to the theory of games.* New York: McGraw-Hill, 1952.

Meek, B. I. The formulation of models of party competition. *British Journal of Political Science*, 1972, *2*, 116-120.

Merkl, P. Coalition politics in West Germany. In S. Groenings, E. W. Kelly, & M. M. Leiserson (Eds.), *The study of coalition behavior.* Toronto: Holt, Phinehart & Winston, 1970.

Merton, R. *Social theory and social structure.* Glenco, Illinois: The Free Press, 1957.

Mesarovic, M. D., & Takahara, Y. *General systems theory: Mathematical foundations.* New York: Academic Press, 1975.

Monna, A. F. *Functional analysis in historical perspective.* New York: John Wiley & Sons, 1973.

Murakami, Y. *Logic and social choice.* London: Routledge & Kegan Paul, 1968.

Nagel, E. *Logic without metaphysics.* Glencoe, Ill.: Free Press, 1956.

Nagel, E. *The structure of science.* New York: Harcourt, Brace & World, 1961.

Nagel, E. *Teleology revisited.* New York: Columbia University Press, 1979.

Niemi, R. H., & Weisberg, H. F. (Eds.). *Probability models of collective decision-making.* Columbus, Ohio: Charles E. Merrill, 1972.

Northrop, F. S. C. *The logic of the sciences and the humanities.* New York: Macmillan, 1947.

Ordeshook, P. The spatial theory of elections: a review and a critique. *School of urban and public affairs, reprint no. 180.* Pittsburgh: Carnegie Mellon University.

Pareto V. *Manual of political economy* (A. A. Schwier, trans.). New York: A. M. Kelley, 1971. (Originally published, 1909.)

Pareto, V. *The mind and society* (A. Bongiorno & A. Livingston, trans.). London: Jonathon Cape, 1935. (Originally published, 1916.)

Parsons, T. *The structure of social action.* New York: The Free Press, 1937.

Parsons, T., & E. A. Shils (Eds.), *Toward a general theory of action.* New York: Harper, 1951.

Parsons, T. *Societies.* Englewood Cliffs: Prentice:Hall, 1966.

Penrose, L. H. The elementary statistics of majority voting. *Journal of the Royal Statistical Society* (Series A), 1946, *109*, 53-57.

Petrov, I. P. *Variational methods in optimum control theory* (M. Friedman, trans.) London: Academic Press, 1968.

Pierce, A. Durkheim and functionalism. In K. H. Wolff (Ed.), *Essays on sociology and philosophy.* New york: Harper & Row, 1960.

Pinard, M. *The rise of a third party.* Englewood Cliffs, N. J.: Prentice-Hall, 1971.

Qualter, T. H. An application of the cube law to the Canadian electoral system. *Canadian Journal of Political Science*, 1968, *1* (3), 336-344.

Rae, D. W. *The political consequences of electoral laws.* London: Yale University Press, 1967.

Rae, D. W. Decision-rules and individual values in constitutional choice. *American Political Science Review*, 1969, *63*, 42-56.

Rapoport, A. Uses and limitations of mathematical models in social science. In L. Gross (Ed.), *Symposium on sociological theory.* London: Harper & Row, 1959.

Rapoport, A. *Two-person game theory: the essential ideas.* Ann Arbour: University of Michigan Press, 1966.

Rapoport, A. Forward. In W. Buckley (Ed.), *Modern systems research for the behavioral scientist.* New York: Aldine, 1968.

Rapoport, A. *N-person game theory: Concepts and applications.* Don Mills: Longmans Canada, 1970.

Rapoport, A. & Guyer, M. A taxonomy of 2X2 games. *General Systems,* 1966, *11,* 203-214.

Riker, W. H. *The theory of political coalitions.* New Haven: Yale University Press, 1962.

Riker, W. H., & Ordeshook, P. *An introduction to positive political theory.* Engelwood Cliffs, N. J.: Prentice-Hall, 1973.

Rosenthal, H. Election simulations. *European Journal of Sociology,* 1965, *6,* 21-42.

Rosenthal, H. Political coalitions: elements of a model and the study of French legislative elections. *Proceedings of the International Symposium on Mathematical and Computer Methods in the Social Sciences,* 1967, vol and pp.

Rosenthal, H. Voting and coalition models in election simulations. In W. D. Coplin (Ed.), *Simulation in the study of politics.* Chicago: Markham, 1968.

Samuels, W. J. *Pareto on policy.* London: Elsevier, 1974.

Sartori, G. European political parties: the case of polarized pluralism. In J. Palombara & M. Wiener

(Eds.), *Political parties and political development* Princeton: Princeton University Press, 1966.

Schelling, T. C. *The strategy of conflict.* Oxford: Oxford University Press, 1963.

Schofield, N. The theory of dynamic games. In Ordeshook, P. C. (Ed.), *Game theory and political science.* New York: New York University Press, 1978.

Schumpeter, J. A. *History of economic analysis.* New York: Oxford Univrsity Press, 1964.

Sen, A. K. *Collective choice and social welfare.* London: Hoden-Day, 1970.

Shapley, L. S., & Shubik, M. A method for evaluating the distribution of power in a committee system. *American Political Science Review*, 1954, *48* , 787-792.

Shepsle, K. Parties, voters, and the risk environment: A mathematical treatment of electoral competition under uncertainty. In Niemi, R. H., & Weisberg, H. F. (Eds.), *Probability models of collective decision-making.* Columbus, Ohio: Charles E. Merrill, 1972.

Smith, D. E. (Ed.). *Sourcebook in mathematics.* New York: McGraw-Hill, 1929.

Sommerhoff, G. *Analytical biology.* London: Oxford University Press, 1950.

Sommerhoff, G. Abstract characteristics of living systems. In E. F. Emery (Ed.), *Systems thinking.* London: Penguin, 1969.

Sommerhoff, G. *Logic of the living brain.* London: John

Wiley & Sons, 1974.

Spafford, D. Electoral systems and voter's behavior. *Comparative Politics*, 1972, *5*, 129-134.

Spafford, D. *Seats and votes in plurality elections when more than two parties contend.* Unpublished manuscript, Universty of Saskatchewan, 1975.

Steen, L. A. *Scientific American*, 1980, *243*.

Stevens, S. S. The quantification of sensation. *Daedalus*. 1959, *88*.

Stinchcombe, A. L. *Constructing social theories.* New York: Harcourt, Brace & World, 1968.

Stinchcombe, A. L. *Theoretical models in social history.* New York: Academic Press, 1978.

Stokes, D. E, Spatial models of party competition. *American Political Science Review*, 1963, *57*, 368-377.

Suppes, P. *Introduction to logic.* Toronto: van Nostrand, 1957.

Sztompka, P. *System and function: toward a theory of society.* New York: Academic Press, 1974.

Tauber, S., & White, H. *Systems Analysis.* Toronto: Saunders, 1969.

Theil, H. The desired political entropy. *American Political Science Review* 1969, *63*, 521-525.

Thomson, D. (Ed.) *Political ideas.* Harmondsworth, England: Penquin, 1966. Science Review, 1969, *63*, 521-25.

Von Neumann, J., & Morgenstern, O. *Theory of games and economic behavior.* New York: John Wiley & Sons, 1949.

Wiener, N., Rosenblueth, A. & Bigelow, J. Behavior, purpose and teleology. *Philosophy of science.* 1943, *10* .

Whetstone, A. W. The bare-majority decision rule in the two alternative case. Unpublished master's thesis, University of Saskatchewan, 1974.

Woodfield, A. *Teleology.* Cambridge: Cambridge University Press, 1976.

Wright, L. *Teleological explanations.* Berkeley: Univrsity of California Press, 1976.

Zahler, R. S., & Sussmann, H. J. Claims and accomplishments of applied catastrophe theory. *Nature*, 1977, *269* (Oct. 27).